Sag-Tension Computations and Field Measurements of Bonneville Power Administration

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ANY TRULY flexible material of uniform weight, which a conductor approximates, will conform to a catenary when suspended between two supports. However, many forms of sagging computations are based on the simpler parabolic equation or on a compromise between the catenary and the parabolic equation as shown for some of the basic catenary equations in Fig. 1. For example, the value of y (level span sag) for a catenary may be written as:

$$y = H/w\left(\cosh\frac{wx}{H}-1\right) \text{ or } \frac{wx^2}{2H}+\frac{w^3x^4}{24H^3}+\ldots+\frac{w^{(2n-1)}x^{2n}}{(2n!)H^{(2n-1)}}+\ldots$$

The first term in the series is a parabolic sag based on uniform weight along the chord similar to the cable loading on a suspension bridge. The first two terms of the series give a close approximation of the true y value as expressed by either the hyperbolic or exponential form of the catenary equation. In all cases, the sag from the series equation will be less than that for a true catenary. It is interesting to note that any inflexibility in a cable will also result in a sag somewhat less than the catenary sag.

For heavy loading areas with comparatively low stringing tensions in long spans, and particularly in steep inclined spans, the difference between the catenary and the parabolic curve can be appreciable. Thus, it is important to know the limitations of methods used in computing sags and tensions in the office and in making measurements in the field.

This matter takes on additional importance in view of trends toward reduced clearances and less "built-in" protection as reflected in proposed National Electric Safety Code (NESC) revisions for high voltage transmission.

With the service area of the Bonneville Power Administration (BPA) in the Pacific Northwest extending from the Rocky Mountains across two major mountain ranges to the Pacific Ocean, a considerable portion of our over 8,000 miles of transmission lines is constructed in some of the most rugged mountain terrain in the United States and in areas of severe icing. BPA became exposed very early in its construction program to the problems of stringing large conductors over steep, inclined spans at relatively low tensions. As a result, there evolved methods of sag-tension calculations and field measurements that are a blending and simplification of old methods.

This paper will describe the BPA method of computing sag-tension data and also some of the controls used in applying these data to the field.

Sag-Tension Computations

A conductor, whether suspended between towers or supported on the ground, will change in length due to changes in temperature and tension. At constant temperature and within the elastic limit, its change in length, ΔL, for a given change in tension, ΔT, follows Hooke's law:

$$\Delta L = \frac{L\Delta T}{(\text{Area})(\text{Modulus of Elasticity})}$$

Hooke's law may be graphically illustrated by tension-strain curves. The slope of these curves for copper and aluminum changes with tension within the working limit of the conductor. Furthermore, the slope of these curves for bimetallic conductor such as ACSR (aluminum cable steel reinforced) also changes with temperature.

On the other hand, a conductor of a given uniform weight suspended as a catenary between two towers will also follow the law of the catenary; that is, tension and sag are a function of slack (difference between curve length L and straight line length C between

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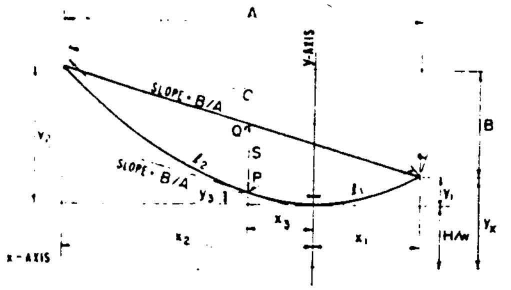


NOMENCLATURE

A, B, C = HORIZONTAL, VERTICAL, AND SLOPE DISTANCE BETWEEN SUPPORTS.

CO-ORDINATES MEASURED FROM POINT OF HORIZONTAL TANGENCY ON CONDUCTOR.

- x, y = ANY POINT
- x_1, y_1 = LOWER SUPPORT
- x_2, y_2 = UPPER SUPPORT
- x_3, y_3 = SAG POINT

y_x = ORDINATE MEASURED FROM DIRECTRIX (x-AXIS)

ℓ = CONDUCTOR LENGTH MEASURED FROM LOW POINT TO ANY POINT x, y

$L = \ell_1 + \ell_2$ = CONDUCTOR LENGTH BETWEEN SUPPORTS

S = SAG = DISTANCE IN VERTICAL PLANE MEASURED FROM LINE BETWEEN SUPPORTS TO THE POINT ON THE CONDUCTOR WITH B/A SLOPE

H = HORIZONTAL COMPONENT OF CONDUCTOR TENSION

T = CONDUCTOR TENSION AT ANY POINT ON CATENARY

w = WEIGHT OF CONDUCTOR PER UNIT LENGTH

H/w = VALUE OF y_x WHEN $x = 0$

PRECISION METHOD

(BY USE OF HYPERBOLIC FUNCTIONS)

ITEM NO

1 $y_x = \frac{H}{w}\cosh\frac{x}{H/w}$

2 $y = y_x - \frac{H}{w} = \frac{H}{w}\left(\cosh\frac{x}{H/w} - 1\right)$

3 $\ell = \frac{H}{w}\sinh\frac{x}{H/w}$

4 SLACK = L − A FOR LEVEL SPAN = $2(\ell - x)$

SLACK = L − C FOR INCLINED SPAN = $\ell_1 + \ell_2 - C$

5 $T = H\cosh\frac{x}{H/w} = H + H\cosh\frac{x}{H/w} - H$

$= H + w\left(\frac{H}{w}\cosh\frac{x}{H/w} - \frac{H}{w}\right) = H + wy$

APPROXIMATE METHOD

(ITEMS 1 AND 3 EXPANDED BY MACLAURIN'S SERIES)

ITEM NO

1. $y_x = \frac{H}{2w}\left(e^{\frac{wx}{H}} + e^{\frac{-wx}{H}}\right) = \frac{H}{w}\left[1 + \frac{\left(\frac{wx}{H}\right)^2}{2!} + \frac{\left(\frac{wx}{H}\right)^4}{4!} + \cdots\right.$

$= \frac{H}{w} + \frac{wx^2}{2H} + \frac{w^3x^4}{24H^3} + \cdots$

2. $y = y_x - \frac{H}{w} = \frac{wx^2}{2H} + \frac{w^3x^4}{24H^3} + \cdots$

3. $\ell = \frac{H}{2w}\left(e^{\frac{wx}{H}} - e^{\frac{-wx}{H}}\right) = \frac{H}{w}\left[\frac{wx}{H} + \frac{\left(\frac{wx}{H}\right)^3}{3!} + \frac{\left(\frac{wx}{H}\right)^5}{5!} + \cdots\right.$

$= x + \frac{w^2x^3}{6H^2} + \frac{w^4x^5}{120H^4} + \cdots$

4. SLACK = L − A FOR LEVEL SPAN (REF. 2)

$= \frac{w^2A^3}{24H^2} + \frac{w^4A^5}{1920H^4} + \cdots$

SLACK = L − C FOR INCLINED SPAN (REF. 2)

$= \frac{w^2A^4}{24CH^2} + \frac{w^4A^6}{1920CH^4} + \cdots$

5. $T = H + wy$

PRECISION METHOD

(BY USE OF HYPERBOLIC FUNCTIONS)

ITEM NO

6 $B = y_2 - y_1 = \frac{H}{w}\left(\cosh\frac{[A - x_1]}{H/w} - \cosh\frac{x_1}{H/w}\right)$

AND FROM $\cosh N - \cosh M = \left(2\sinh\frac{M+N}{2}\right)\left(\sinh\frac{M-N}{2}\right)$

$x_1 = \frac{A}{2} - \frac{H}{w}\sinh^{-1}\frac{B/2}{\frac{H}{w}\sinh\frac{A/2}{H/w}}$

7. $x_2 = \frac{A}{2} + \frac{H}{w}\sinh^{-1}\frac{B/2}{\frac{H}{w}\sinh\frac{A/2}{H/w}}$

8 $\frac{dy}{dx} = \sinh\frac{x}{H/w}$

9 S: (a) DETERMINE x_3 DISTANCE TO POINT "P" ON CURVE WHERE SLOPE = B/A USING ITEM 8

(b) DETERMINE y_3 AND ELEVATION "P" FOR THE VALUE OF x_3 USING ITEM 2

(c) DETERMINE ELEVATION POINT "Q"

(d) S = ELEVATION "Q" − ELEVATION "P"

APPROXIMATE METHOD

ITEM NO

6 $B = y_2 - y_1 = \frac{w(A - x_1)^2}{2H} - \frac{wx_1^2}{2H}$

OR $x_1 = \frac{A}{2S}\left(S - \frac{B}{4}\right)$

7 $x_2 = \frac{A}{2S}\left(S + \frac{B}{4}\right)$

8 $\frac{dy}{dx} = \frac{wx}{H} + \frac{w^3x^3}{6H^3} +$

SUBSTITUTING ITEM 6 OR 7 FOR x AND USING $\frac{wx}{H}$ PART OF SERIES ONLY, THEN

(a) APPROXIMATE SLOPE AT LOWER SUPPORT =

$w/H\left[\frac{A}{2S}\left(S - \frac{B}{4}\right)\right] = \frac{wA}{8HS}(4S - B) = \frac{(4S-B)}{A}$

(b) APPROXIMATE SLOPE AT UPPER SUPPORT = $\frac{(4S+B)}{A}$

9 $S = \frac{wAC}{8H} + \frac{3A^2 - 2B^2}{144C}\left(\frac{A^3w^3}{8H^3}\right)$ (REF. 2)

Fig. 1. Basic catenary equations

supports). This law may be graphically illustrated by catenary function curves in which per-cent slack, $100(L-C)/L$, is plotted as abscissa with tension and sag as ordinate.

A most convenient and accurate method of computing sags and tensions for a conductor suspended as a catenary for different loadings and temperatures is a semigraphical method. In this method the catenary function curves for a given span length are superimposed over the tension-strain curves of the conductor plotted for each temperature concerned. This method is used by BPA and is similar to that used by Varney[1] for level spans and Ehrenburg[2] for steep inclined spans.

The BPA method involves the use of new catenary function tables for level dead end (DE) spans. These tables and other simplifications reduce computations and possible errors.

Tension-Strain Curves

These curves are developed from repeated tension-strain test data taken at approximate stranding temperature for the conductor concerned. For ACSR conductor, tension-strain curves are required for both the complete conductor and the steel core. The procedure summarized here is similar to that shown in reference 1.

As shown in Fig. 2, with per cent strain ($100\Delta L/L$) plotted as abscissa and tension in pounds as ordinate, certain loads are held for a period of time to stabilize the conductor. A large percentage (tests showed 90% in one case) of the elongation occurring during these holding periods is caused by strand setting. The remainder is creep. After each hold period the conductor is unloaded and then reloaded. A curve drawn through the points at the end of each hold period is called the virtual initial tension-strain curve. The duration of hold periods shown in Fig 2 is not necessarily adequate for all types of stranding or material. However, later field prestressing (tension and time) for sagging is correlated by BPA with the elongation represented by this virtual initial tension-strain curve.

For an ACSR conductor, the steel core is subject to the same type of repeated loading. The steel core is loaded to match the initial elongation of the ACSR conductor at each hold period and then held for the same period of time. For example, at the first hold period the steel core is stressed in pounds to equal Ta_sE_s/aE. With elongation, ΔL, and sample length, L, made the same for both the ACSR conductor and the steel core stress-strain tests, then

$$\Delta L = \frac{\Delta TL}{aE} = \frac{\Delta T_sL}{a_sE_s}$$

$$T_s = \frac{Ta_sE_s}{aE}$$

for T and T_s initially equal to zero

T and T_s = tension for conductor and steel core

E and E_s = modulus of elasticity for conductor and steel core

a and a_s = cross-section area for conductor and steel core

Plotting both the ACSR conductor and the steel core data to the same scale as in Fig. 2, one can obtain the virtual initial and final tension-strain curves for the aluminum strands of the ACSR conductor by direct subtraction.

There are now tension-strain curves for the aluminum and steel components

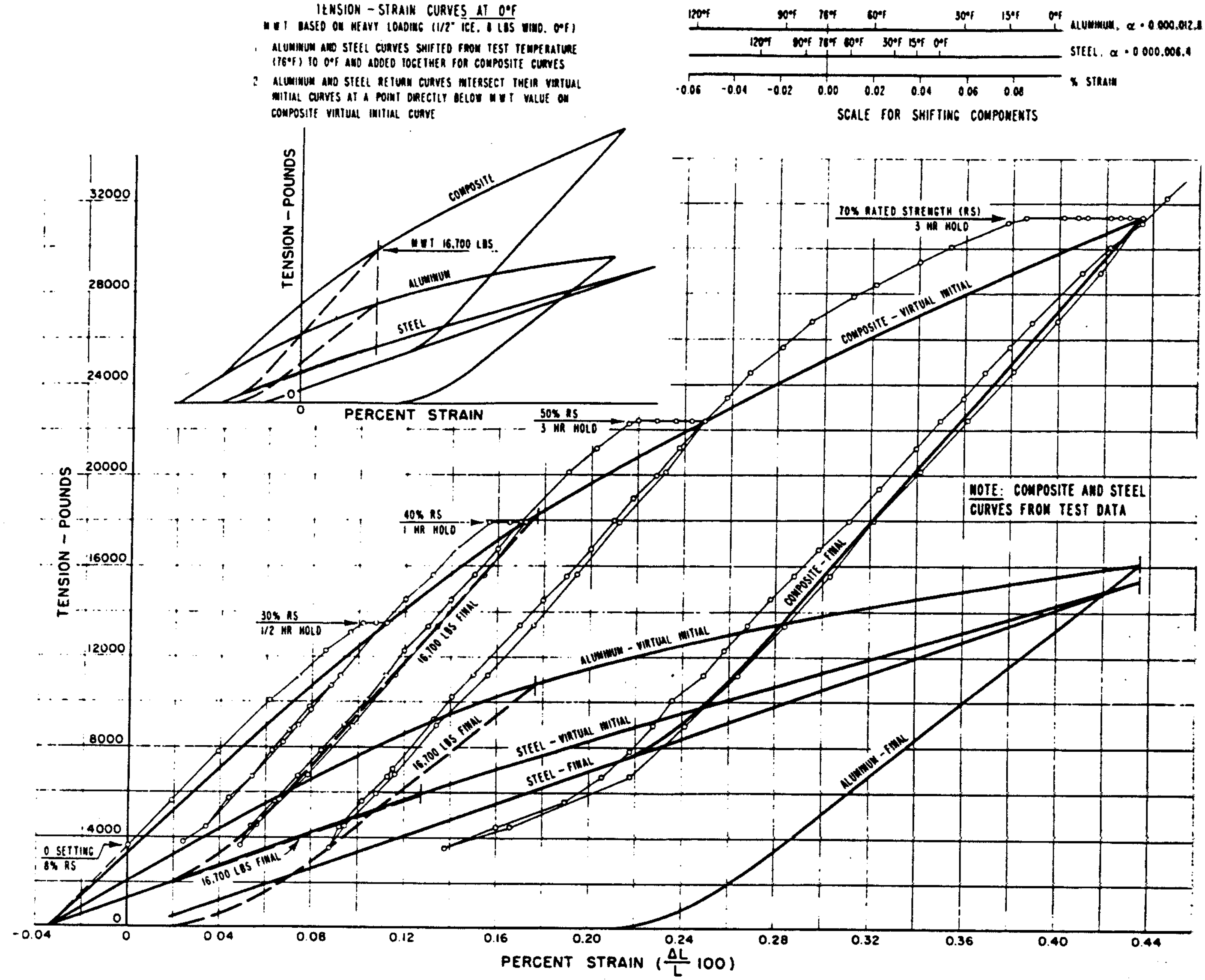


Fig. 2. Tension-strain curves for ACSR Pheasant conductor, rated strength 44,800 pounds, test temperature 76 F

of the ACSR conductor at test temperatures. By shifting the aluminum and steel curves horizontally to right or left by the product of the temperature change desired and the coefficient of expansion of the components and then adding the two shifted curves together, one obtains the tension-strain curves, initial and final, for the complete ACSR conductor for any temperature desired.

Fig. 3 shows tension-strain curves constructed in this manner for ACSR Pheasant conductor at stringing temperatures. Final curves are constructed for maximum working tension (MWT) of 8,000 pounds and 16,700 pounds at 0 F (degrees Fahrenheit). These are the MWT values used by BPA for this conductor for wood pole and steel construction, respectively. Note that the intersection of initial and final curves increases from the MWT value on the 0 F curve to progressively higher values on the 32 F and 120 F curves. This is the result of the aluminum strands taking a larger percentage of the total load at 0 F than at the higher temperatures. Thus, at the higher temperatures the aluminum strands are still at final modulus of elasticity for tensions in excess of MWT.

As shown in the insert on Fig. 2, the intersection of the initial and final curves for MWT value must be first located on the component aluminum and steel curves with the curves shifted horizontally to the temperature for the MWT loading, such as 0 F for heavy loading. Then, in subsequent shifting of components for other temperatures, both the initial and final curves shift together.

Catenary Function Curves

Curves in Fig. 4 give for a certain span length the catenary relationship between per-cent slack plotted as abscissa and sag and tension plotted as ordinate. The catenary function curves are plotted to the same scale as the aforementioned tension-strain curve. Both the slack, $L-C$, of the catenary function curves and the change in conductor length, ΔL, for the tension-strain curves are plotted as abscissa in per cent of conductor length—stressed in the former and unstressed in the latter case. The resulting error from using stressed length is very small, but greatly simplifies the computation of the catenary function curves. A practical scale for the abscissa is:

1 inch = 0.02% slack or 0.02% strain

A tension curve is computed for each type of conductor loading for which sag-tension data are desired. The support tension (T_m) curve is the value of tension in the catenary at the support. The effective tension (T_e) curve is the value of tension for the catenary that

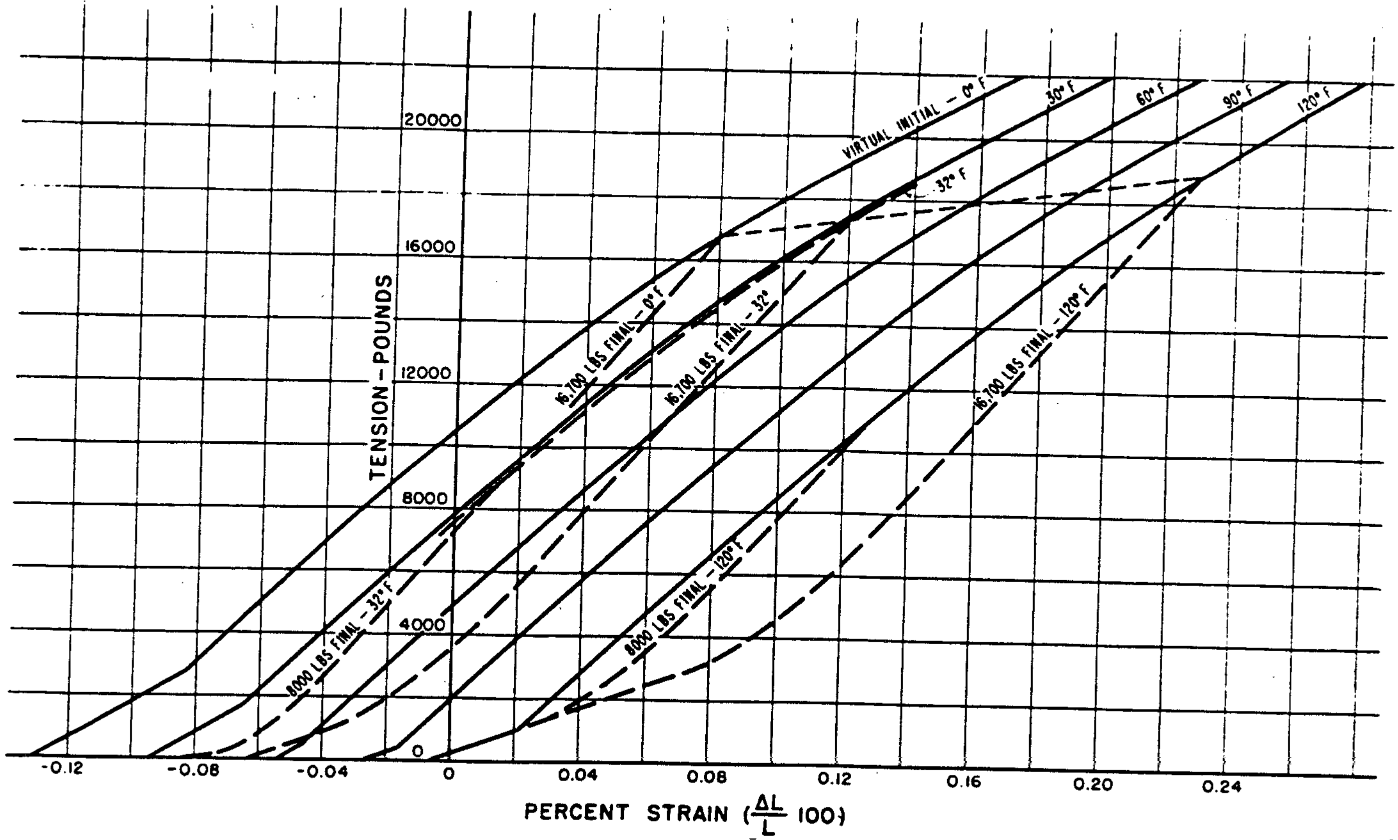
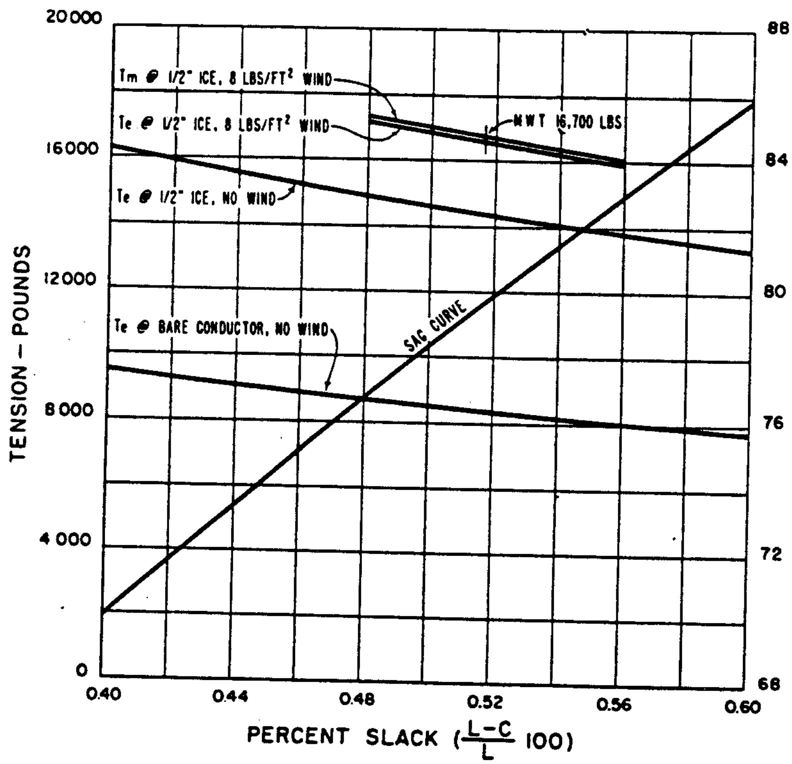


Fig. 3. Tension-strain curves for ACSR Pheasant conductor, stringing temperatures



COMPUTATIONS OF DATA FOR CATENARY FUNCTION CURVES

A = 1800 FT B = 0 C = A

Aw_3 = 5803 LBS Aw_2 = 5051 LBS Aw_1 = 2943 LBS

FOR TENSION AND SAG FACTORS SEE TABLE I

% SLACK (TABLE VALUE)	0.40	0.44	0.48	0.52	0.56	0.60
Tm, MAXIMUM DESIGN LOADING.........LBS (Aw_3) (SUPPORT TENSION FACTOR)			17,316	16,654	16,065	
Tm–Te, MAXIMUM DESIGN LOADING.....LBS (Aw_3) (SUPPORT–EFF TENSION FACTOR)			165	171	178	
Te, ICED CONDUCTOR, NO WIND........LBS (Aw_2) (EFFECTIVE TENSION FACTOR)	16,345	15,588	14,928	14,347	13,829	13,363
Te, BARE CONDUCTOR, NO WIND......LBS (Aw_1) (EFFECTIVE TENSION FACTOR)	9,523	9,083	8,698	8,359	8,057	7,786
SAG...FEET (A) (SAG FACTOR)	69.95	73.39	76.68	79.84	82.88	85.82

Fig. 4. Catenary function curves, ACSR Pheasant conductor, 1,800-foot span

Table I. Catenary Functions

% Slack	Support Tension Factor	Support-Effective Tension Factor	Effective Tension Factor	Sag Factor
0.0005	90.910	0.000	90.910	0.00138
0.001	64.937	0.001	64.936	0.00193
0.002	45.665	0.002	45.663	0.00274
0.004	32.283	0.003	32.280	0.00387
0.006	26.348	0.003	26.345	0.00474
0.008	22.826	0.004	22.822	0.00548
0.010	20.414	0.004	20.410	0.00612
0.015	16.674	0.005	16.669	0.00750
0.020	14.443	0.006	14.437	0.00866
0.025	12.918	0.007	12.911	0.00968
0.030	11.795	0.007	11.788	0.01062
0.040	10.216	0.008	10.208	0.012252
0.050	9.1427	0.0091	9.1336	0.013695
0.060	8.3317	0.0100	8.3217	0.015035
0.080	7.2323	0.0115	7.2208	0.017332
0.100	6.4750	0.0129	6.4621	0.019373
0.120	5.9144	0.0141	5.9003	0.021223
0.160	5.1285	0.0163	5.1122	0.024510
0.200	4.5894	0.0183	4.5711	0.027428
0.240	4.1976	0.0200	4.1776	0.030062
0.280	3.8911	0.0216	3.8695	0.032481
0.320	3.6391	0.0231	3.6160	0.034736
0.360	3.4346	0.0245	3.4101	0.036855
0.400	3.2618	0.0259	3.2359	0.038862
0.440	3.1133	0.0271	3.0862	0.040773

% Slack	Support Tension Factor	Support-Effective Tension Factor	Effective Tension Factor	Sag Factor
0.480	2.9839	0.0284	2.9555	0.042601
0.520	2.8699	0.0295	2.8404	0.044355
0.560	2.7684	0.0306	2.7378	0.046045
0.600	2.6774	0.0317	2.6457	0.047677
0.660	2.5568	0.0333	2.5235	0.050030
0.720	2.4518	0.0348	2.4170	0.052282
0.780	2.3594	0.0362	2.3232	0.054444
0.840	2.2772	0.0376	2.2396	0.056529
0.900	2.2033	0.0389	2.1644	0.058547
0.960	2.1368	0.0401	2.0967	0.060494
1.02	2.07628	0.04146	2.03482	0.062388
1.08	2.02097	0.04268	1.97829	0.064230
1.14	1.97017	0.04387	1.92630	0.066024
1.20	1.92332	0.04502	1.87830	0.067781
1.26	1.87992	0.04615	1.83377	0.069484
1.32	1.83959	0.04725	1.79234	0.071156
1.38	1.80199	0.04833	1.75366	0.072793
1.44	1.76683	0.04938	1.71745	0.074397
1.50	1.73386	0.05042	1.68344	0.075970
1.56	1.70286	0.05144	1.65142	0.077515
1.62	1.67365	0.05243	1.62122	0.079033
1.68	1.64607	0.05341	1.59266	0.080525
1.74	1.61999	0.05438	1.56561	0.081993
1.80	1.59526	0.05532	1.53994	0.083438
1.86	1.57178	0.05626	1.51552	0.084861

% Slack	Support Tension Factor	Support-Effective Tension Factor	Effective Tension Factor	Sag Factor
1.92	1.54945	0.05718	1.49227	0.086264
1.98	1.52819	0.05808	1.47011	0.087647
2.04	1.50791	0.05898	1.44893	0.089012
2.10	1.48854	0.05986	1.42868	0.090359
2.16	1.47002	0.06073	1.40929	0.091688
2.22	1.45229	0.06159	1.39070	0.093001
2.28	1.43530	0.06244	1.37286	0.094299
2.34	1.41899	0.06327	1.35572	0.095582
2.40	1.40333	0.06410	1.33923	0.096850
2.48	1.38339	0.06519	1.31820	0.098520
2.56	1.36444	0.06627	1.29817	0.100166
2.64	1.34640	0.06732	1.27908	0.101791
2.72	1.32922	0.06837	1.26085	0.103394
2.80	1.31282	0.06940	1.24342	0.104977
2.88	1.29714	0.07041	1.22673	0.106541
2.96	1.28215	0.07142	1.21073	0.108086
3.04	1.26780	0.07241	1.19539	0.109614
3.12	1.25403	0.07339	1.18064	0.111126
3.20	1.24083	0.07436	1.16647	0.112621
3.30	1.22505	0.07556	1.14949	0.114468
3.40	1.21002	0.07674	1.13328	0.116292
3.50	1.19570	0.07790	1.11780	0.118094
3.60	1.18203	0.07906	1.10297	0.119875
3.70	1.16896	0.08019	1.08877	0.121636
3.80	1.15646	0.08132	1.07514	0.123378

% Slack	Support Tension Factor	Support-Effective Tension Factor	Effective Tension Factor	Sag Factor
3.90	1.14448	0.08243	1.06205	0.125102
4.00	1.13300	0.08353	1.04947	0.126809
4.10	1.12198	0.08461	1.03737	0.128498
4.20	1.11140	0.08569	1.02571	0.130171
4.30	1.10123	0.08676	1.01447	0.131830
4.40	1.09145	0.08782	1.00363	0.133473
4.50	1.08203	0.08886	0.99317	0.135102
4.60	1.07295	0.08989	0.98306	0.136717
4.70	1.06420	0.09092	0.97328	0.138319
4.80	1.05576	0.09193	0.96383	0.139908
4.90	1.04762	0.09294	0.95468	0.141485
5.00	1.03975	0.09394	0.94581	0.143050
5.10	1.03214	0.09493	0.93721	0.144604
5.20	1.02479	0.09592	0.92887	0.146146
5.30	1.01767	0.09689	0.92078	0.147678
5.40	1.01078	0.09786	0.91292	0.149199
5.50	1.00411	0.09882	0.90529	0.150711
5.60	0.99765	0.09978	0.89787	0.152212
5.70	0.99139	0.10073	0.89066	0.153704
5.80	0.98531	0.10167	0.88364	0.155188
5.90	0.97942	0.10260	0.87682	0.156662
6.00	0.97370	0.10353	0.87017	0.158128

satisfies Hooke's law. For example, if the unstressed length of a conductor suspended as a catenary is stressed uniformly throughout its length by the T_e value for the catenary, the conductor will elongate to the same length as that of the conductor suspended in the catenary. T_e may be considered the average tension acting on the catenary length of the conductor and is always less than $(T_m + H)/2$.

The catenary function tables developed by BPA provide a direct, simple and accurate method of determining the data for plotting the catenary function curves for any level DE span.

Derivation of Equations for Catenary Function Tables

Precise equations in Fig. 1 may be written for the level span in Fig. 5 as follows:

$$\text{Sag} = y = H/w\left(\cosh \frac{wx}{H} - 1\right) \qquad (1)$$

$$\text{Total conductor length, } L = 2\ell = \frac{2H}{w} \sinh \frac{wx}{H} \qquad (2)$$

$$\text{Support tension, } T_m = T = H \cosh \frac{wx}{H} \qquad (3)$$

The length of a small element of unstressed conductor $(\Delta\ell_u)$ when suspended becomes[2]

$$\Delta\ell = \Delta\ell_u\left(1 + \frac{T}{aE}\right)$$

for conductor of cross-section area (a) and modulus of elasticity (E).

$$\ell = \int_0^{\ell_u} d\ell_u + \frac{1}{aE}\int_0^{\ell_u} T d\ell_u = \ell_u + \frac{1}{aE}\int_0^{\ell_u} T d\ell_u.$$

or

$$\frac{\frac{1}{\ell_u}\int_0^{\ell_u} T d\ell_u}{\ell - \ell_u} = \frac{aE}{\ell_u} = \frac{T_e}{\ell - \ell_u}$$

As this is an expression of Hooke's law and the effective tension is the value of tension that satisfies Hooke's law,

$$T_e = \frac{1}{\ell_u}\int_0^{\ell_u} T d\ell_u$$

or very nearly

$$T_e = \frac{1}{\ell}\int_0^{\ell} T d\ell$$

where ℓ is the length of stressed cable in the catenary

Substituting equations 2 and 3 and integrating:

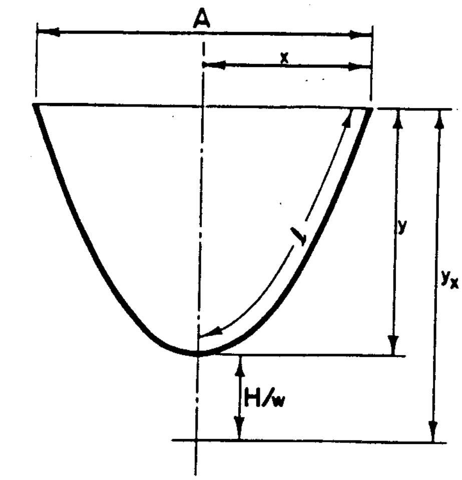


Fig. 5. Level span catenary

Conductor length between supports: $2\ell = L$

$$T_e = \frac{H}{\ell}\int_0^x \cosh^2\frac{wx}{H}\,dx$$

$$= \frac{H^2}{2w\ell}\left(\sinh\frac{wx}{H}\cosh\frac{wx}{H} + \frac{wx}{H}\right) \quad (4)$$

Slack $= 2(\ell - x)$

$$\%\ \text{Slack} = \frac{2(\ell - x)100}{2\ell} = \left(1 - \frac{x}{\ell}\right)100$$

or using equation 2

$$\%\ \text{Slack} = 100\left(1 - \frac{wx}{H\sinh\frac{wx}{H}}\right)$$

let

$$\frac{wx}{H} = Z$$

then

$$\%\ \text{Slack} = 100\left(1 - \frac{Z}{\sinh Z}\right) = 100 - \frac{100Z}{\sinh Z}$$

also

$$\frac{\sinh Z}{Z} = \frac{1}{1 - \frac{\%\ \text{Slack}}{100}} \quad (5)$$

Development of Catenary Function Tables

By use of hyperbolic tables, the value of Z corresponding to each per-cent slack selected for the first column of Table I is found from equation 5 by trial and error method and interpolation. The National Bureau of Standards, United States Department of Commerce, publishes 9-place "Tables of Circular and Hyperbolic Sines and Cosines for Radian Arguments" that are excellent to use for these computations.

Having calculated the Z value for each selected per-cent slack, it is possible to derive sag and tension factors applicable to any span length by expressing equations 1, 3, and 4 in terms of the then known values of Z and isolating the unknowns (span length A and conductor weight w). Thus,

$$\text{sag} = y = \frac{x}{Z}(\cosh Z - 1)$$

from equation 1

$$\text{sag} = \frac{A}{2Z}(\cosh Z - 1)$$

$$\text{sag factor} = \frac{\text{sag}}{A} = \frac{1}{2Z}(\cosh Z - 1) \quad (6)$$

$$\text{support tension, } T_m = Aw\frac{(\cosh Z)}{2Z}$$

from equation 3

$$\text{support tension factor} = \frac{T_m}{Aw} = \frac{\cosh Z}{2Z} \quad (7)$$

$$\text{effective tension, } T_e = Aw\left[\frac{\cosh Z}{4Z} + \frac{1}{4\sinh Z}\right]$$

from equations 2 and 4

effective tension factor

$$= \frac{T_e}{Aw} = \frac{\cosh Z}{4Z} + \frac{1}{4\sinh Z} \quad (8)$$

If the support tension factor is determined first from equation 7, the effective tension factor can be more conveniently found by expressing equation 8 as follows:

effective tension factor

$$= \frac{1}{4Z}\left[2Z(\text{support tension factor}) - \frac{\%\ \text{slack}}{100} + 1\right] \quad (9)$$

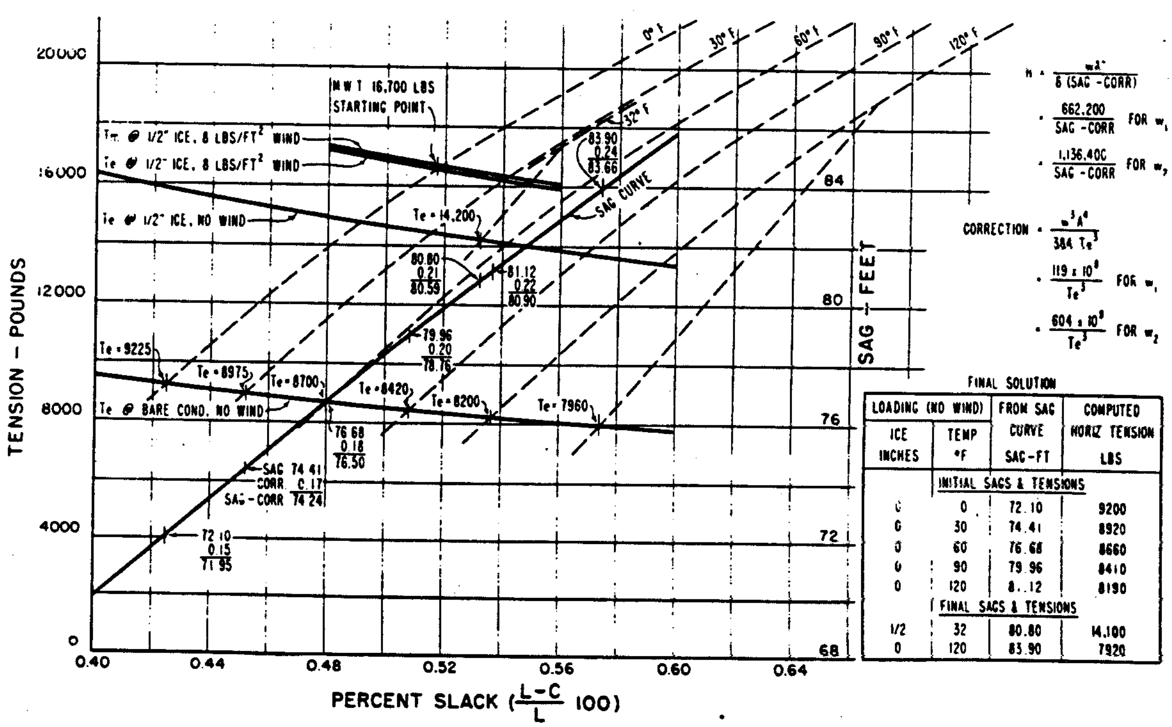
Horizontal tension may be computed as follows:

$$H = Aw\,(\text{support tension factor} - \text{sag factor}) \quad (10)$$

Or from approximate equation under item 2 in Fig. 1,

$$\text{level span sag} = y = \frac{wA^2}{8H} + \frac{w^3A^4}{384H^3} \quad (11)$$

then



FINAL SOLUTION

LOADING (NO WIND) ICE INCHES	TEMP °F	FROM SAG CURVE SAG-FT	COMPUTED HORIZ TENSION LBS
	INITIAL SAGS & TENSIONS		
0	0	72.10	9200
0	30	74.41	8920
0	60	76.68	8660
0	90	79.96	8410
0	120	8..12	8190
	FINAL SAGS & TENSIONS		
1/2	32	80.80	14,100
0	120	83.90	7920

Fig. 6. Fig. 4 superimposed over Fig. 3

Table II. Table for Finding Horizontal Tension from Sag

$\frac{A}{8\text{ Sag}}$	q	$\frac{A}{8\text{ Sag}}$	q	$\frac{A}{8\text{ Sag}}$	q	$\frac{A}{8\text{ Sag}}$	q
13.889		2.766		1.522		1.037	
	0.002		0.008		0.014		0.020
8.327		2.437		1.414		0.983	
	0.003		0.009		0.015		0.021
5.947		2.178		1.319		0.934	
	0.004		0.010		0.016		0.022
4.622		1.967		1.236		0.890	
	0.005		0.011		0.017		0.023
3.779		1.793		1.162		0.849	
	0.006		0.012		0.018		0.024
3.195		1.647		1.096		0.811	
	0.007		0.013		0.019		0.025
2.766		1.522		1.037		0.775	

$$\frac{H}{Aw}=\frac{A}{8(\text{Sag}-\text{Correction})}=\frac{A}{8\text{ Sag}}+q$$

Then $H=(Aw)\left(\frac{A}{8\text{ Sag}}+q\right)$

Compute $\frac{A}{8\text{ Sag}}$ from known sag and span length. Locate the tabulated $\frac{A}{8\text{ Sag}}$ values next higher and next lower than the computed value. Between these two is listed the quantity, q, which should be added to the computed $\frac{A}{8\text{ Sag}}$ to give horizontal tension factor $\frac{H}{Aw}$. For instance, if $\frac{A}{8\text{ Sag}}=2.529$, $q=0.008$.

Then $\frac{H}{Aw}=2.537$ or $H=2.537\ Aw$.

$$H=\frac{wA^2}{8(\text{sag}-\text{correction})} \qquad (12)$$

The correction is small; a close approximation of this correction $w^3A^4/384T_e^2$. The then computed value of H can be quickly used to check the approximation.

In summarizing, Table I is computed from equations 6, 7, and 9 after finding values of Z from equation 5 for selected values of per-cent slack.

Calculating Catenary Function Curves From Table I

For successive table values of per-cent slack, the following computations are required for a given level span length:

1. $T_m=Aw$(support tension factor)
2. $T_m-T_e=Aw$(support-effective tension factor)
3. $T_e=Aw$(effective tension factor)
4. Sag$=A$(sag factor)

The first two are usually only for the maximum design loading. The T_e curve for maximum design loading is most accurately plotted by subtracting the T_m-T_e values, step 2, from the T_m value. A T_e curve, step 3, is computed for each type of conductor loading for which sag-tension data are required, such as bare conductor at 0 wind for stringing charts, and ice and wind loadings or combinations thereof for clearance and overload studies. The value of w must correspond to the total weight per unit length of the conductor for loading conditions concerned. Only one sag curve is required for a given span.

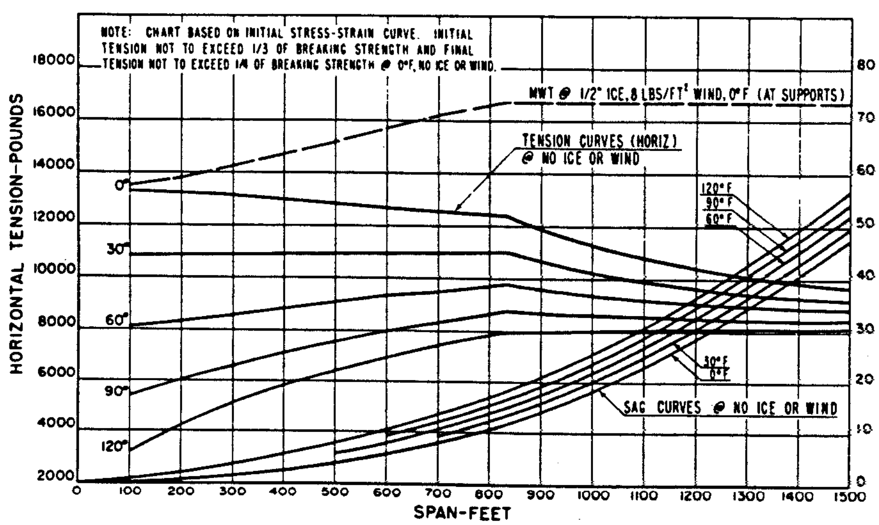


Fig. 7. Stringing chart—dead end spans

For the usual problem where a given MWT is the starting point, three points are sufficient for the T_m and T_e curves at maximum design loading. One uses only the point on the former curve that intersects the MWT ordinate and the corresponding point on the T_e curve directly below this intersection (see Fig. 6).

The MWT starting point, as shown on Fig. 6, can actually be computed without plotting the two maximum design loading curves if the per-cent slack corresponding to MWT can be determined with reasonable accuracy. One such convenient means is by use of Martin's tables.[8] Knowing MWT, one calculates Aw/MWT and interpolates for the length factor; then % Slack$=100$ (length factor-1)/length factor, for MWT starting point on T_m curve.

The T_m-T_e value computed from Table I by interpolation for this calculated per-cent slack will locate T_e for maximum design loading. This method of determining the MWT starting point is particularly more accurate when the maximum design loading curves are relatively flat.

The number of computations required for the bare T_e curve and the sag curve can be easily determined after working several spans. Usually five to six points are adequate.

Sag-Tension Computations for Level DE Spans

For a given MWT at specified design loading, the catenary function curves, Fig. 4, are superimposed over the tension-strain curves, Fig. 3, as shown in Fig. 6. With the tension scales aligned, the catenary function curves are shifted horizontally until the initial tension-strain curve for MWT temperature intersects the T_e curve for maximum design loading at a point directly below the MWT starting point on the T_m curve.

As the effective tension satisfies Hooke's law, it follows that the intersection of the tension-strain curve for any temperature with any T_e curve will determine the effective tension and per-cent slack in the span for the respective temperature and loading concerned. The sag is read off the sag curve at a point vertically below or above this intersection. For other than short spans, the horizontal tension can be most accurately determined from the known sag by equation 12 or by Table II. In Table II, equation 12 is written in the form of

$$\frac{H}{Aw}=\frac{A}{8(\text{sag}-\text{correction})}=\frac{A}{8\text{ sag}}+q$$

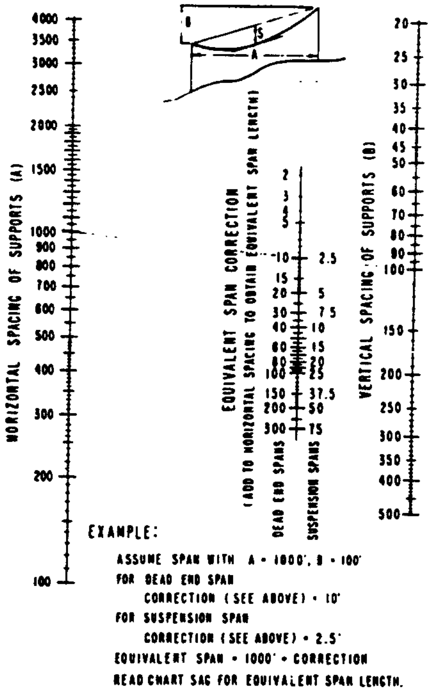


Fig. 8. Nomograph for determining level span equivalent of nonlevel spans

Fig. 9. Derivation of field sagging equation

Let P represent point of tangency which line of sight makes with catenary and S_2 represent the difference between S and S_1; then S_2 is the sag of span PQ and the slope of $\phi = \frac{4S_2}{PQ}$ (item 8, Fig. 1)

$$S - S_1 = S_2$$

$$S + S_2 = \frac{h+t}{2}$$ midline of trapezoid that bisects nonparallel sides equals one half the sum of the two parallel sides

transpose and solve for S_2

$$S_2 = \frac{h+t}{2} - S$$

also $\tan \phi = \frac{4S_2}{PQ} = \frac{h-t}{A}$

$$\frac{\overline{PQ}^2}{A^2} = \frac{S_2}{S} = \frac{(4S_2)^2}{(h-t)^2}$$ square of span lengths proportional to sags

Substitute $S - S_1$ for S_2 and simplify

$$\frac{S-S_1}{S} = \frac{16(S-S_1)^2}{(h-t)^2}$$

$$S - S_1 = \frac{(h-t)^2}{16S} = S_2$$

then $\frac{(h-t)^2}{16S} = \frac{(h-t)}{2} - S$

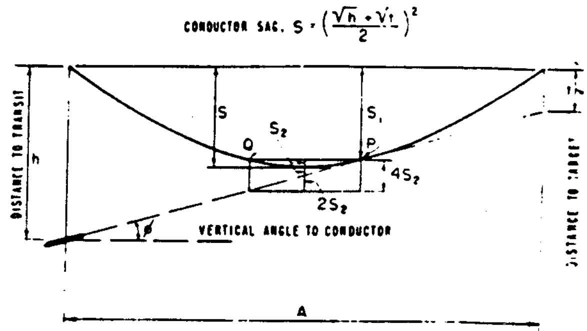
$$(h-t)^2 = 8S(h+t) - 16S^2$$

Solve quadratic equation at completing the square

$$[4S-(h+t)]^2 = (h+t)^2 - (h-t)^2$$

$$[4S-(h+t)] = 2\sqrt{ht}$$

$$S = \left(\frac{\sqrt{h}+\sqrt{t}}{2}\right)^2$$



For short spans in the order of 500 feet or less, where a considerable change in tension results in a very small change in sag, it is more accurate to calculate sag by equation 12 from the known tension (T_e). For short spans, T_e and H are equal for all practical purposes and the sag correction in equation 12 is negligible.

Example:

Given: ACSR Pheasant conductor
MWT = 16,700 pounds at 1/2 inch ice, 8 pounds per square foot wind, 0 F
Span length = 1,800 feet
Conductor weight in pounds per foot:

$w_1 = 1.635$ for bare conductor, no wind

$w_2 = 2.806$ for 1/2 inch ice, no wind

$w_3 = 3.224$ for 1/2 inch ice, 8 pounds per square foot wind

Find: 1. Initial stringing sags and horizontal tensions at:

0 F, 30 F, 60 F, 90 F, 120 F

2. Final sags and horizontal tensions at:

1/2 inch ice, no wind, 32 F
0 inch ice, no wind, 120 F

For computations of data and the catenary function curves plotted from these data, see Fig. 4. For the remainder of the computations necessary for final solution, see Fig. 6.

Sag-Tension Computations for Steep Inclined DE Spans

Data for catenary function curves are computed by the Ehrenburg method.[2] The T_m at upper support, T_e, and sag values of the inclined catenary are plotted against per-cent slack. These curves are then used with the tension-strain curves in the same manner as in the example for level span computations.

In addition to its simplicity and accuracy, the method just illustrated provides a flexible means of analyzing many of the incidental conductor problems related to new construction, line maintenance, and line revisions such as prestressing, crossing clearances, conductor overloads, effects of creep, etc. The tension-strain curves and catenary function curves accumulated in the preparation of stringing charts for new construction become valuable tools for subsequent design problems.

Field Measurements

Ruling Span

As shown by DE chart in Fig. 7, the change in tension for a given temperature change varies with span length. This can be explained by the slack-tension relations of a catenary. Where DE spans are at approximately the same horizontal tension, a given per-cent change in slack causes an increasing change in tension as span lengths become shorter. Now, in a series of suspension spans that have been sagged to the same horizontal tension with all suspension strings plumb, the tension in each span, regardless of length, will change essentially the same with changes in temperature due to small movements of the suspension strings along line. The suspension strings swing to decrease the slack change in short spans and increase the slack change in long spans. This movement of the suspension string is so small that the horizontal force exerted by the string on the conductor is normally negligible.

A DE span that gives the same change in tension from changes in loading and temperature as that in a series of suspension spans between two anchor or DE towers is called the ruling span of the series of suspension spans. The ruling span equation is based on the slack-tension relations of n' spans, each of ruling span length (A_r), matching the slack-tension relations of a number of unequal spans of the same total length. Thus, by equating total length,

$$n'A_r = A_1 + A_2 + \ldots + A_n$$

Using the first term of approximate slack equation for a level span from Fig. 1 and equating slack for a given horizontal tension (H):

$$\frac{n'w^2A_r^3}{24H^2} = \frac{w^2(A_1^3 + A_2^3 + \ldots + A_n^3)}{24H^2}$$

$$n'A_r^3 = A_1^3 + A_2^3 + \ldots + A_n^3$$

Substituting for $n'A_r$:

$$(A_1 + A_2 + \ldots + A_n)A_r^2 = A_1^3 + A_2^3 + \ldots + A_n^3$$

$$\text{ruling span} = A_r = \sqrt{\frac{A_1^3 + A_2^3 + \ldots + A_n^3}{A_1 + A_2 + \ldots + A_n}}$$

Where the spans in a series of suspen-

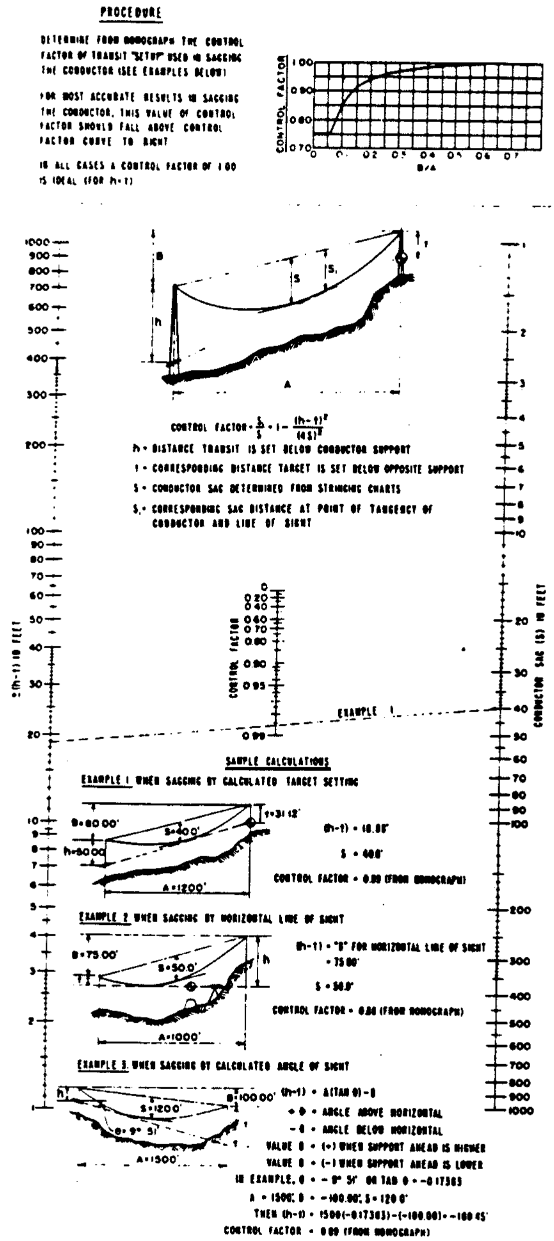


Fig. 10 (left). Nomograph for determining control for conductor sagging

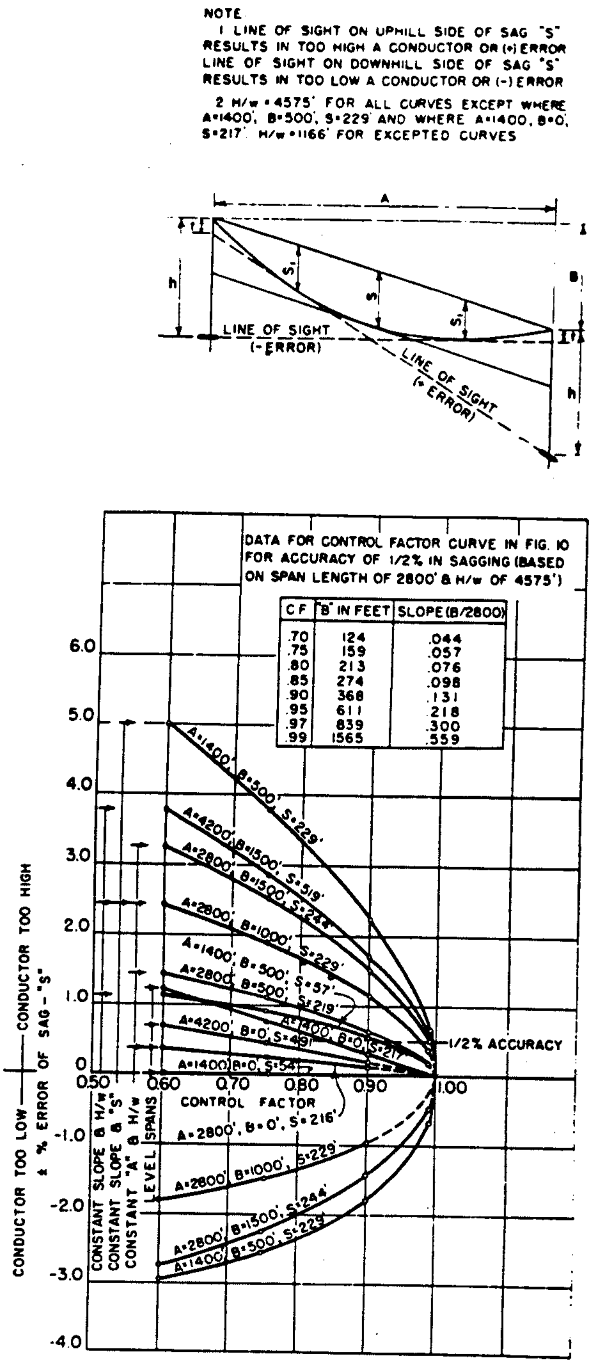


Fig. 11 (right). Control factor versus sagging error

sion spans vary considerably in length, the tension in the extremely long and short spans will not be fully equalized to that of the ruling span and considerable error can result in design clearances. As shown by the tension curves of the DE chart in Fig. 7, the extremely long spans will have greater cold-weather sag and smaller hot-weather sag than anticipated; the opposite will be true for the short spans. Clearances to both ground and overhead crossings can be affected. The problem can be avoided by dead ending.

Equivalent Span

Stringing charts give sags and horizontal tensions for sagging level spans. For nonlevel spans, the sag is obtained from the stringing chart for an equivalent level span. The equivalent level span is equal to:

1. $\sqrt{AC}$ or approximately $A+B^2/4A$ for suspension spans.
2. $2C-A$ from reference 3, or approximately $A+B^2/A$ for DE spans.

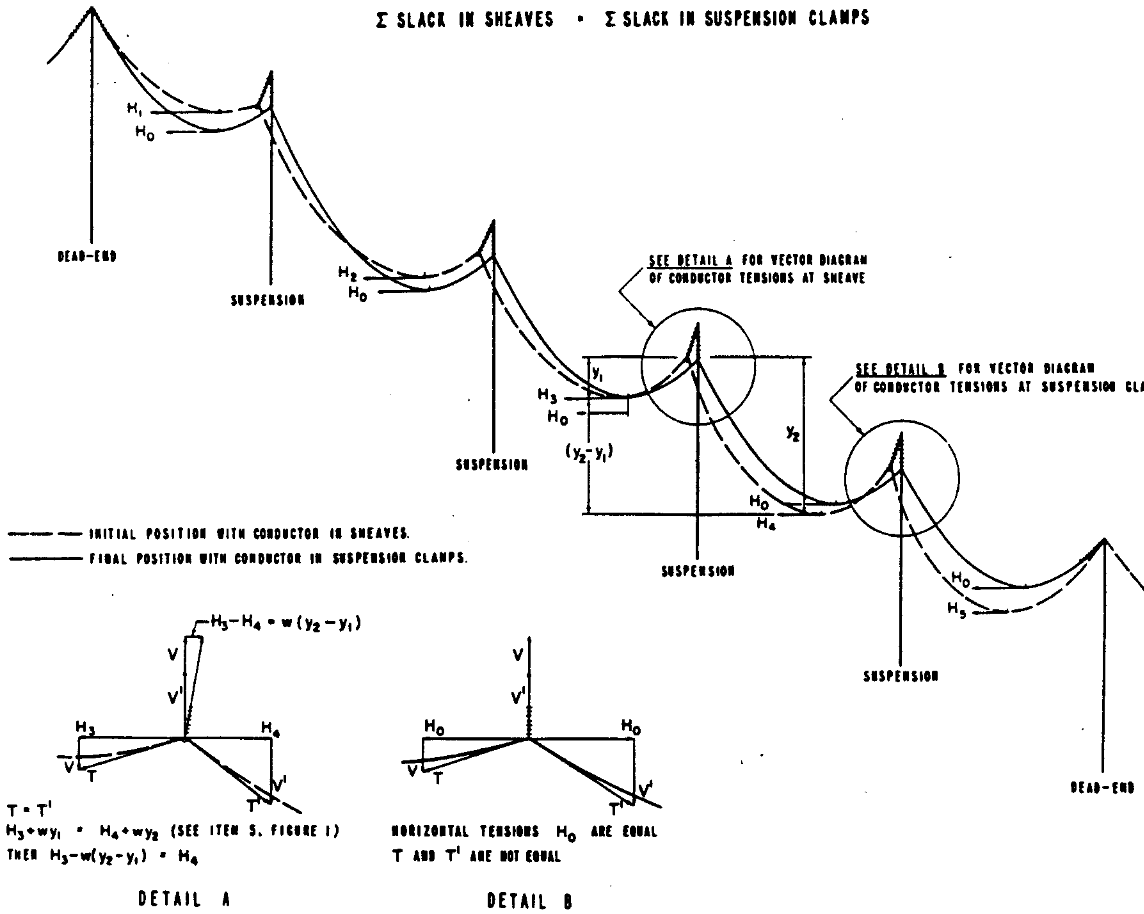
These approximations are based on substituting $A+B^2/2A$ for C. Note that the approximate equation for the equivalent level DE span gives four times the correction to be added to span length (A) as is required for the equivalent level suspension span.

The nomograph shown in Fig. 8 is constructed from the approximate forms of the equivalent level span equations and provides a convenient method for determining the value of the equivalent span in the field.

Theory of Equivalent Level Span Equations

To assure plumb suspension strings at each supporting tower, the conductor in each span of a series of suspension spans must be sagged for a given temperature at the same horizontal tension regardless of span length (A) or difference in support elevations (B). The sags for ruling span charts accomplish this for level spans as these sags for a given temperature are based on the same horizontal tension for any span length. Now,

Fig. 12. Graphical analysis of insulator offsets and sag corrections



the approximate equation for the sag of a nonlevel span under item 9 in Fig. 1, is

$$S=\frac{wAC}{8H}+\left(\frac{3A^2-2B^2}{144C}\right)\left(\frac{A^2w^2}{8H^2}\right) \text{ or}$$

$$\frac{wAC}{8H}+\text{correction} \quad (13)$$

This can be written as

$$S=\frac{w(\sqrt{AC})^2}{8H}+\text{correction}$$

The difference of this correction for a level and a nonlevel span is negligible. The correct sag and horizontal tension for a nonlevel span is then obtained from the ruling span chart by looking up the sag for the equivalent level span, $\sqrt{AC}$. Stringing chart sags include the correction for level spans.

Level spans sagged by ruling span charts will have a support tension at maximum design loading equal to MWT in spans of ruling span length. For longer or shorter spans the support tension at maximum design loading is respectively greater or smaller than MWT value. On the other hand, DE charts are designed to give a support tension in level spans equal to the MWT value for maximum design loading regardless of span length. As a result, the stringing tension for a given temperature changes in a DE chart with span length. The equivalent level span $(2C-A)$ provides the additional sag required in a nonlevel DE span to approximate the MWT value at the upper support for maximum design loading. It also permits the use of Table I for computing the catenary function curves to determine the sags of an inclined span. However, the correct horizontal tensions for these sags are not those shown on the DE chart or by the catenary function curves, but can be computed from equation 13.

DE spans on extremely steep hillsides may be computed by the Ehrenburg method[2] and the stringing data submitted to the field in tabulation form.

Sagging Equation

The conductor is strung to the correct sag by the commonly used field sagging equation derived in Fig. 9.

$$S=\left(\frac{\sqrt{h}+\sqrt{t}}{2}\right)^2$$

for checking sag; or

$$t=(2\sqrt{S}-\sqrt{h})^2$$

for target shots; or

$$\tan\phi=\frac{h-t+B}{A}$$

for angle shots in which B is (+) or (—) when support ahead is higher or lower and ϕ is (+) or (−) when angle is above or below the horizontal.

Errors of Field Sagging Equation

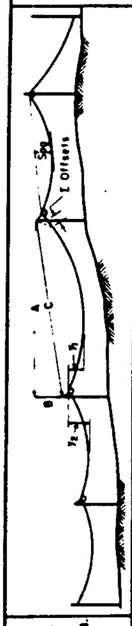
The field sagging equation is based on a parabola and involves several approximations when applied to a catenary. With reference to Fig. 9, these are:

1. Slope $(4S_1/PQ)$ is approximate for catenary.
2. Sag (S) in a catenary lies toward upper support from midpoint in an inclined span.
3. Sags of catenaries are only approximately proportional to the square of span lengths.

The only theoretically accurate sag made by the field sagging equation is one in which $h=t$ or the control factor (S_1/S) is equal to 1.00 (see Fig. 10). Fig. 11 shows that the error in sagging a given span increases as the point of tangency of the line of sight on a conductor approaches either support from true sag position $(h=t)$ or as the control factor decreases. Furthermore, the sagging error for a given span changes in direction when the point of tangency of the line of sight is on the uphill or downhill side of the true sag position. The curves in Fig. 11 also show that the sagging error increases with increasing span length and slope

Nomenclature:

y_2-y_1 = Difference of elevation of low points of sag in adjacent spans from maximum sag profile.
H_o = Horizontal tension of conductor in clamps.
H_e = Equivalent horiz. tension of conductor in sheaves.
H = Assumed " " " " " "
H' = Corrected " " " " " "



w = 0.546 lbs/ft, w/2 = 0.273
$w^2/2$ = 0.1491
H_o = 1622 lbs at 60° F
$10^6/(H_o)^2$ = 0.38010
$H_e = \sqrt{\Sigma(8)/\Sigma(10)} \times 10^3$ = 1750
$H_o - H_e$ = −128

Conductor ACSR "IBIS"
M.W.T. 5000 lbs at 1/2 - 8 - 0
Ruling Span 800 ft.
Stringing Temp. 60° F.

U S DEPARTMENT OF THE INTERIOR
BONNEVILLE POWER ADM.
INSULATOR OFFSETS & SAG CORRECTIONS
LONG FORM

LINE OLYMPIA - ABERDEEN NO. 1
SECTION 12/5 — 14/2

BY ____ DATE ____ CHKD BY ____ DATE ____

Offset of Suspension Clamp from Point below Support																		Sag of Conductor in Sheaves						Correction in Sheaves	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26
Tower No	(A) Span Length Feet	(B) Diff. in Elev Feet	(C) Inclined Length Feet	y_2-y_1 Feet	$w(y_2-y_1)$ Pounds	Assumed H Pounds	$\frac{A^4}{10^6C}$	$\frac{H^2}{10^6}$	$\frac{A^4}{CH^2}$	H' Pounds	$\frac{w^2A^4}{2\times10^6C}$	$\frac{10^6}{(H')^2}$	$\frac{10^6}{(H')^2}-\frac{10^6}{(H_o)^2}$	Offset Inches	Final Adjustment Inches	Final Offset Inches	Σ Offsets Inches	$\frac{C}{4}$	$\frac{A}{48}$	$Z=\frac{wA}{2H}$	$\frac{CZ}{4}$	$\frac{AZ^3}{48}$	Sag Feet	Chart Sag Feet	Sag Correction Feet
Parenthesized numbers refer to column numbers →			$\sqrt{(2)^2+(3)^2}$		w(5)		$\frac{(2)^4}{10^6(4)}$	$\frac{(7)^2}{10^6}$	$\frac{(8)}{(9)}$	$(7)+H_o-H_e$	$\frac{w^2}{2}(8)$	$\frac{10^6}{(11)^2}$	$(13)-\frac{10^6}{(H_o)^2}$	(12)(14) & Mod Corr	$\frac{\Sigma(15)}{\Sigma(12)}(12)$	(15)+(16)	Σ(17)	$\frac{(4)}{4}$	$\frac{(2)}{48}$	$\frac{w(2)}{2(11)}$	(19)(21)	$(20)(21)^3$	(22)+(23)	Use Equiv. Span	(24)−(25)
12/5																	0								
	450	3	450			2055	91	4.2	21.7	1927	13.57	0.26930	−0.11080	−1.50	+0.04	−1.88	−	112.5	9	0.064	7.2		7.2	8.5	−1.3
12/6				−34	−19									−0.42			−2								
	490	47	492			2036	117	4.1	28.5	1908	17.44	0.27469	−0.10541	−1.84	+0.05	−2.23	−	123.0	10	0.070	8.6		8.6	10.1	−1.5
12/7				−56	−31									−0.44			−4								
	690	51	692			2005	328	4.0	82.0	1877	48.90	0.28384	−0.09626	−4.71	+0.15	−5.11	−	173.0	14	0.100	17.3		17.3	20.1	−2.8
12/8				−86	−47									−0.55			−9								
	745	89	750			1958	411	3.8	108.2	1830	61.28	0.29861	−0.08149	−4.99	+0.19	−5.28	−	187.5	16	0.111	20.8		20.8	23.6	−2.8
13/1				−97	−53									−0.48			−15								
	650	98	657			1905	272	3.6	75.6	1777	40.56	0.31668	−0.06342	−2.57	+0.12	−2.77	−	164.3	14	0.100	16.4		16.4	18.0	−1.6
13/2				−117	−64									−0.32			−17								
	865	117	873			1841	641	3.4	188.5	1713	95.57	0.34079	−0.03931	−3.76	+0.29	−3.72	−	218.3	18	0.138	30.1		30.1	31.9	−1.8
13/3				−100	−55									−0.25			−21								
	760	102	767			1786	435	3.2	135.9	1658	64.86	0.36377	−0.01633	−1.06	+0.20	−0.95		191.8	16	0.125	24.0		24.0	24.7	−0.7
13/4				−40	−22									−0.09			−22								
	355	38	357			1764	44	3.1	14.2	1636	6.56	0.37362	−0.00648	−0.04	+0.02	−0.04		39.3	7	0.059	5.3	—	5.3	5.2	+0.1
13/5				−55	−30									−0.02			−22								
	870	36	871			1734	658	3.0	219.3	1606	98.11	0.38771	+0.00761	+0.75	+0.30	+1.09		217.8	18	0.148	32.2	0.1	32.3	31.9	+0.4
13/6				+9	+5									+0.04			−21								
	855	59	857			1739	624	3.0	208.0	1611	93.04	0.38531	+0.00521	+0.48	+0.29	+0.80		214.3	18	0.145	31.1	0.1	31.2	30.8	+0.4
13/7				+21	+11									+0.03			−20								
	360	44	363			1750	46	3.1	14.8	1622	6.86	0.38010	0	0	+0.02	+0.02		90.8	8	0.061	5.5	—	5.5	5.4	+0.1
14/1				−192	−105									0			−20								
	1322	178	1334			1645	2290	2.7	848.1	1517	341.44	0.43454	+0.05444	+18.59	+1.05	+20.07		333.5	28	0.238	79.4	0.4	79.8	74.6	+5.2
14/2														+0.43			0								
							5957		1944.8		888.19		Σ = −2.72	−0.65 / −2.07	$\frac{\Sigma(15)}{\Sigma(12)}$ = −0.00306										

NOTES

Modulus Correction $= \frac{12C(H_e-H')}{aE_{FM}}$ inches

$\frac{aE_{FM}}{12}$ = 323,070

a = Cross section area for conductor

E_{FM} = Final modulus of elasticity

Offsets in direction of first tower listed are positive.

Fig. 13. Insulator offsets and sag corrections, long form

(*B*/*A*) and with decrease in *H*/*w*. This error can be appreciable for low stringing tensions on steep inclined spans.

This analysis of the sagging error was made as a result of complaints from the field that the conductor was not properly sagging in some rough terrain in the Cascade Mountains. Even though insulator offsets and sag corrections were correctly used, the conductor in one sagging span would tend to go high and in another sagging span low, etc., when sagging several spans in a series of suspension spans.

The resulting analysis showed the importance of controlling the sagging setup. The nomograph in Fig. 10 provides such a means of control. This nomograph is constructed from equation:

$$\text{control factor} = \frac{S_1}{S} = 1 - \frac{(h-t)^2}{(4S)^2}$$

This is derived directly from the following equation taken from Fig. 9:

$$S - S_1 = \frac{(h-t)^2}{16S}$$

or

$$S_1 = S - \frac{(h-t)^2}{16S}$$

By means of the nomograph, the "sagger" can check in a moment the adequacy of his transit setup as this control factor is determined by data already available to the sagger. See sample calculations in Fig. 10.

The control factor curve in Fig. 10, used to control the adequacy of the sag, is plotted from the intersection of the error curves (for 2,800-foot span and *H*/*w* of 4,575 feet only) in Fig. 11 with the 1/2% accuracy line. Thus, a sagging setup giving a control factor that falls on this curve results in a sagging error of 1/2% for span length and *H*/*w* value noted. In most cases the maximum error would be considerably less than 1/2%, as seldom would a sagger crowd this control factor curve and, furthermore, span lengths would normally be shorter than 2,800 feet. Note also that the break in the curve limits the control factor to 0.75. A control factor curve can be plotted for any accuracy limit desired and for any span length and *H*/*w* value desired. However, the single control factor curve in Fig. 10 has proved adequate.

Insulator Offsets and Sag Corrections

A conductor strung in rough terrain over frictionless sheaves tends to run downhill, that is, the conductor will come to equilibrium with more than chart sag in the lower spans and less than chart sag in the upper spans. The stringing sheaves will always swing toward the span with the horizontal tangency of the conductor, actual or projected, at the higher elevation. This normally is toward the uphill side.

As shown graphically by Fig. 12, the difference in horizontal tension in two adjacent spans with the conductor at rest in sheaves is equal to $w(y_2 - y_1)$. This is also the horizontal component of tension (see $H_3 - H_4$ in vector diagram in Fig. 12, Detail *A*) that causes the insulator string to swing off plumb along line. Furthermore, the horizontal tension in each span becomes less as one strings downhill and more as one strings uphill.

With the conductor properly attached to the suspension clamps and at chart sag in each span, the difference in horizontal tension (H_o) between adjacent spans will be equal to zero and the suspension strings will hang plumb along line. This condition of equilibrium could only have been accomplished in a given number of suspension spans between DE towers or temporary snubs if the correct length of conductor had been previously sagged into the sag section when the conductor was in the sheaves. Expressed mathematically in Fig. 12:

$$\Sigma \text{ slack in sheaves} = \Sigma \text{ slack in suspension clamps}$$

This is the key equation in computing insulator offsets and sag corrections. From an assumed horizontal tension in the first span in the sag-section, one can compute the horizontal tension and the resulting slack in each successive span for the conductor in the sheaves. If the summation of these slacks figured for the conductor in the sheaves is greater or less than that figured for the conductor in the suspension clamps (with horizontal tension from stringing chart the same in each span), then a higher or lower horizontal tension should be used for the second and final slack computations. Both the long and short form methods give the correct value of H for each span after the first assumption. In most cases, computations made within 20 F of sagging temperature are sufficiently accurate. Small changes in sagging temperature have little effect on the result. The maximum sag profile is sufficiently accurate for determining the value of $y_2 - y_1$, regardless of sagging temperature.

Long Form Method

The procedure previously outlined is that followed by the long form method in Fig. 13. Horizontal tension for conductor in sheave is assumed in Column *7* and corrected to H' in Columns *8* to *11*. In the process, one is basically equating the summation of slack in the sheaves to that in the suspension clamps. The equivalent tension (H_e) is the tension that would give the same total slack for the spans in the sag-section as that calculated for each span by the assumed values of H in Column *7*. This total slack in terms of $2(\text{slack})/w^2$ is calculated in Column *10*.

The slack difference in each span for the H' minus H_o value of horizontal tension is determined in Columns *12* to *15* by using the first term of the approximate slack equation in Fig. 1 for an inclined span, $w^2A^4/24CH^2$, converted to inches. This difference in slack in each span is called an offset. Column *15* also makes allowances for changes in conductor length, based on the modulus of elasticity, that occur when the tension in each span is changed from H' to H_o. Column *16* makes final adjustment to zero for summation of individual offsets in each span. The final insulator offset to be measured in the field at each tower is a summation given in Column *18* of the final individual offsets computed for each span in Column *17*.

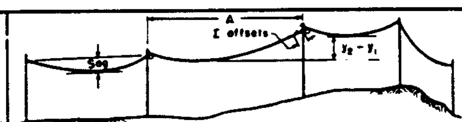
The sags for the conductor in sheaves are computed in Columns *19* to *24* by equation 13 with the correction simplified to that for a level span. This correction is made equal to $w^3A^4/384\,H^3$ by equating B to zero and C to A. The sag corrections given to the field are computed in Column *26* and are the differences between the sags with the conductor in

Nomenclature

$y_2 - y_1$ = Difference of elevation of low points of sag in adjacent spans from maximum sag profile.

H_o = Horizontal tension of conductor in clamps.

H = " " " " " sheaves.



U. S. DEPARTMENT OF THE INTERIOR
BONNEVILLE POWER ADMINISTRATION

INSULATOR OFFSETS & SAG CORRECTIONS
SHORT FORM

LINE: OLYMPIA – ABERDEEN NO. 1
SECTION: 12/5 – 14/2
BY ____ DATE ____ CHK'D BY ____ DATE ____

1	2	3	4	5	6	7	8	9	10	11	12	13	14
Tower No.	(A) Span Length Feet	$y_2 - y_1$ Feet	$w(y_2 - y_1)$ Pounds	Trial $H_o - H$ Pounds	"K" Offset per lb	Trial Offset Inches	Corrected $H_o - H$ Pounds	Corrected Offset Inches	Modulus Correction Inches	Final Adjustment Inches	Final Offset Inches	Σ Offsets Inches	Correction for Sag while in Sheaves, Ft
Parenthesized numbers refer to column numbers →			$w(3)$	$\Sigma(4)$	$\frac{w^2A^3}{H_o^3}$	$(5)(6)$	$\frac{\Sigma(7)}{\Sigma(6)} - (5)$	$(6)(8)$	$\frac{12(2)(8)}{aE_{fm}}$	$-\frac{\Sigma(9)+\Sigma(10)}{\Sigma(6)}(6)$	$(9)+(10)+(11)$	$\Sigma(12)$	$\frac{H_o(9)}{8w(2)}$
12/5												0	
	450			0	0.0064	0	−290	−1.9	−0.4	0	−2.3		−1.6
12/6		−34	−19									−2	
	490			−19	0.0081	−0.15	−271	−2.2	−0.4	0	−2.6		−1.7
12/7		−56	−31									−5	
	690			−50	0.0229	−1.15	−240	−5.5	−0.5	+0.1	−5.9		−3.0
12/8		−86	−47									−11	
	745			−97	0.0284	−2.75	−193	−5.5	−0.4	+0.1	−5.8		−2.7
13/1		−97	−53									−17	
	650			−150	0.0192	−2.88	−140	−2.7	−0.3	+0.1	−2.9		−1.5
13/2		−117	−64									−20	
	865			−214	0.0452	−9.67	−76	−3.4	−0.2	+0.2	−3.4		−1.5
13/3		−100	−55									−23	
	760			−269	0.0303	−8.15	−21	−0.6	0	+0.1	−0.5		−0.3
13/4		−40	−22									−23	
	355			−291	0.0030	−0.87	+1	0	0	0	0		0
13/5		−55	−30									−23	
	870			−321	0.0460	−14.77	+31	+1.4	+0.1	+0.2	+1.7		+0.6
13/6		+9	+5									−22	
	855			−316	0.0436	−13.78	+26	+1.1	+0.1	+0.1	+1.3		+0.5
13/7		+21	+11									−20	
	360			−305	0.0031	−0.95	+15	0	0	0	0		0
14/1		−192	−105									−20	
	1322			−410	0.1615	−66.22	+120	+19.4	+0.5	+0.5	+20.4		+5.4
14/2												0	
					0.4177	−121.34	$\frac{\Sigma(7)}{\Sigma(6)} = 290$	+0.1	−1.5	$\frac{\Sigma(9)+\Sigma(10)}{\Sigma(6)} = -3.35$			

LINE DATA

Conductor: ACSR "IBIS"
M.W.T. 5000 lbs. at 1/2-8-0
Ruling Span: 800 ft.
Stringing Temp.: 60 °F
w = 0.546 lbs./ft.
H_o = 1622 lbs. at 60 °F.
$H_o/8w$ = 371
$(aE_{fm}/12)$ = 323,070

NOTES:

1. Offsets in direction of first tower listed are positive.
2. Sag corrections are to be added algebraically to the chart sag to obtain the sag while in the sheaves.
3. If any one of the three conditions given below apply, then offsets should be used:
   a. Individual offset is 2" or greater (column 12)
   b. Summation of individual offsets is 4" or greater (column 13)
   c. Individual sag correction is 1' or greater (column 14)
4. For severe offsets (greater than 15" in column 12) use Fig. 13.
5. "K" may be taken from previously constructed curves with "K" plotted against span length.

Fig. 14. Insulator offsets and sag corrections, short form

OLYMPIA — ABERDEEN LINE I
ACSR "IBIS" CONDUCTOR

NOMENCLATURE

1. ELEVATION DIFFERENCE = DISTANCE IN FEET BETWEEN THE HIGHEST AND LOWEST LOW POINT (HORIZONTAL TANGENCY) OF THE SAG SECTION
2. 2" DIFFERENCE = 2/(wK) = 2/(0.546 K) = 3.663/K
3. w = BARE WEIGHT OF CONDUCTOR IN POUNDS PER FOOT.
4. H_0 = HORIZONTAL TENSION AT 0 ICE, 0 WIND, 60° F. (INITIAL)
5. K = OFFSET PER POUND = $(w^2)(\text{LONG SPAN}^3)/H_0^3$

NOTE: COMPUTE OFFSETS FOR THE SAG SECTION INVOLVED WHEN TWICE THE 2" DIFFERENCE IS EQUAL TO OR LESS THAN THE ELEVATION DIFFERENCE.

NO. OF SPANS	MILES	SAG SECTION	RULING SPAN	LONG SPAN	MAX. WORKING TENSION	ELEV. DIFF. / 2" DIFF.	OFFSETS
12	1.59	12/5 – 14/2	800	1322	5000 AT 1/2 – 8 – 0	748 / 23	YES — WORKED, USE LONG FORM
9	1.07	14/2 – 15/4	800	875	"	298 / 78	YES — WORKED, NO OFFSETS
15	1.60	23/2 – 24/7	600	799	"	75 / 123	NO
3	0.41	24/7 – 25/2	800	1030	"	217 / 48	YES — WORKED, HAS OFFSETS

Fig. 15. Insulator offset analysis

sheaves and those with the conductor in suspension clamps.

Short Form Method

The computations are considerably reduced in the short form method shown in Fig. 14 by the use of the differentiated form of the first term of the level span equations for slack (Fig. 1) and sag, equation 11. Thus,

$$\text{slack} = L - A = \frac{w^2A^3}{24H^2} \text{ feet}$$

or

$$\frac{w^2A^3}{2H^2} \quad \text{inches for level span}$$

$$d(\text{slack}) = -\frac{w^2A^3}{H^3}\,dH$$

$$\Delta(\text{slack}) = K\Delta H \tag{14}$$

Sign change conforms to Note *1*, Fig. 14, and

$$\text{sag} = S = \frac{wA^2}{8H}$$

Substituting S^2 in slack equation for level span,

$$\text{slack} = \frac{32S^2}{A}$$

$$d(\text{slack}) = \frac{64S\,dS}{A} = \frac{8wA\,dS}{H}$$

or

$$\Delta\text{sag} = \frac{H\Delta(\text{slack})}{8wA} \tag{15}$$

K in equation 14 is the change in slack or offset in inches for a given span per pound change in horizontal tension. Therefore, in order to change the tension in a given span by the difference in horizontal tension for the conductor in the suspension clamp and the sheave or $H_o - H$, one changes the slack in inches by $K(H_o - H)$. The offset is computed by this equation in

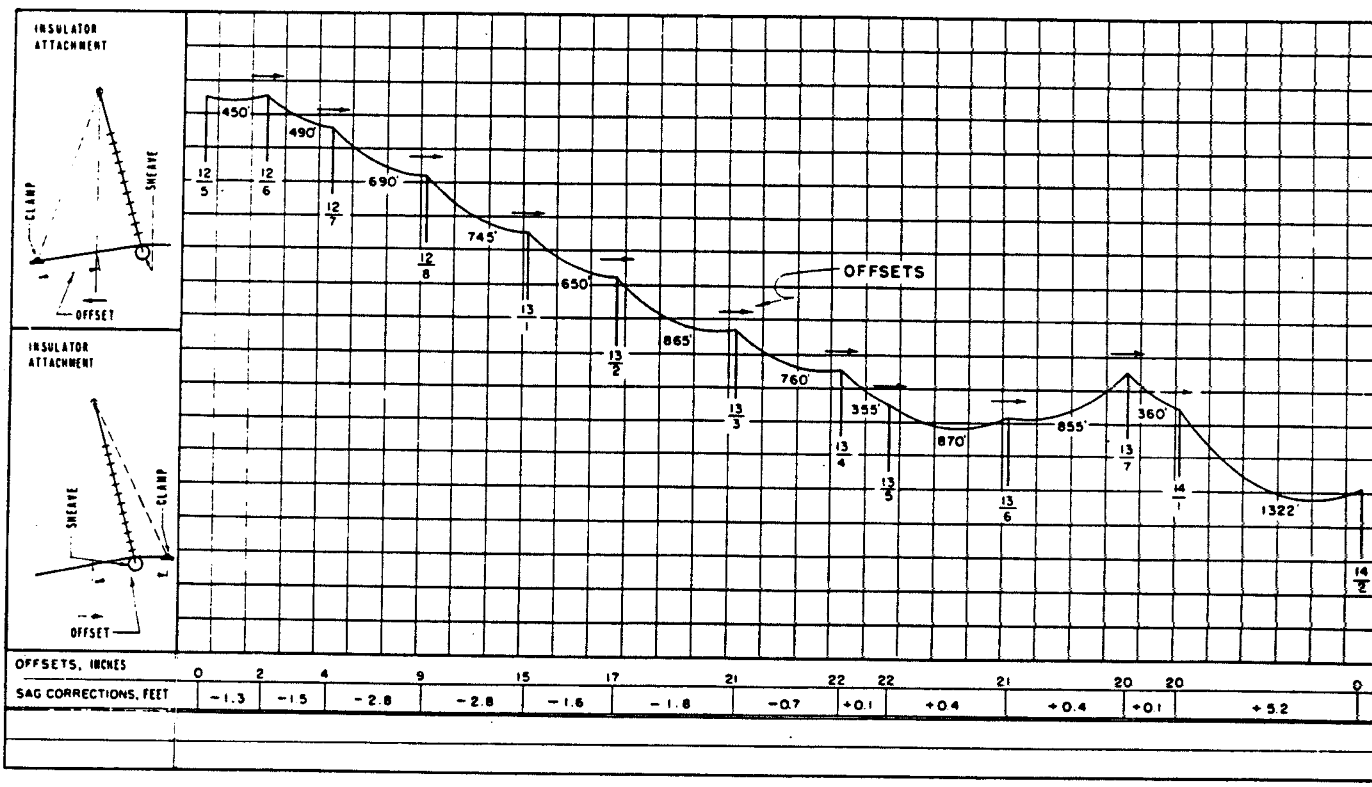


Fig. 16. Insulator offsets and sag corrections, field data

GENERAL: The insulator string on the clipped structure adjacent to new section being sagged should be restrained from longitudinal movement along line when sagging and plumb marking the conductor

OFFSETS are measured from a point vertically below the insulator string attachment in the direction shown by arrows in the above sketch. These offsets are to be marked before any clipping-in

SAG CORRECTIONS are to be added algebraically to the chart sag to obtain the sags while in the sheaves

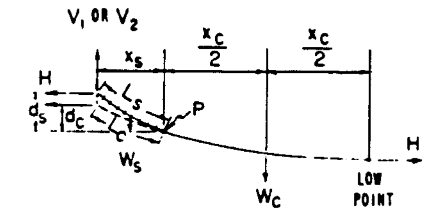


Fig. 17. Insulator sag computations

Nomenclature:

H = Horizontal component of conductor tension
V_1 or V_2 = Vertical load at support
W = Weight of conductor per unit length
W_c = Conductor weight from low point to DE assembly
W_s = DE assembly height
L_s = DE assembly length
x_s = L_s and L_c

For semispan consisting of conductor and dead end assembly with vertical load at support equal to V_1

Σ of vertical forces = 0

$V_1 = W_c + W_s$

Σ of moments about point P = 0

$$V_1x_s - Hd_s - W_s\frac{x_s}{2} = 0$$

$$d_s = \frac{V_1x_s - \frac{W_sx_s}{2}}{H} = \frac{(W_c+W_s)x_s - \frac{W_sx_s}{2}}{H}$$

For dead end assembly in semispan replaced by conductor length L_c with vertical load at support equal to V_2

$$V_2 = W_c + wx_s$$

$$V_2x_s - Hd_c - wx_s\frac{x_s}{2} = 0$$

$$d_c = \frac{V_2x_s - wx_s\frac{x_s}{2}}{H} = \frac{(W_c+wx_s)x_s - wx_s\frac{x_s}{2}}{H}$$

Increase in conductor sag due to weight of dead end assembly = $d_s - d_c$

$$d_s - d_c = \frac{(W_c+W_s)x_s - \frac{W_sx_s}{2}}{H} - \frac{(W_c+wx_s)x_s - wx_s\frac{x_s}{2}}{H}$$

$$d_s - d_c = \frac{x_s}{2H}(W_s - wx_s)$$

Column *7* of the short form for assumed or trial $H_o - H$ and in Column *9* for the corrected $H_o - H$. The process of equating the summation of slack in the sheaves to that in the suspension clamps is accomplished in the short form by correcting $H_o - H$ in Column *8*. Modulus of elasticity correction is calculated in Column *10*. Column *11* makes final adjustment to zero for summation of individual offsets. Column *13* gives the insulator offset to be measured in the field at each tower and is a summation of individual offsets listed in Column *12*.

The sag correction is computed by equation 15 in Column *14*. As differentials are only accurate for changes approaching zero, equations 14 and 15 used in the short form to compute changes in slack and sag are only accurate for small changes in H and slack. The long form should be used for severe offsets. (See Note *4* on short form, Fig. 14.)

Line Analysis

Note *3*, Fig. 14, gives the minimum value of insulator offsets and sag corrections that are sent to the field. The 2-inch limitation for the individual offset is used in Fig. 15 to determine the sections of a transmission line where insulator offsets and sag corrections may be required. Insulator offsets and sag corrections computed for these sections may or may not be sufficient to send to the field. The selections made by Fig. 15 are based on the longest span requiring half the full "Elevation Difference" of the sag-section for a 2-inch offset. Actually, the offset for the longest span of a sag-section is practically always based on less than half this full "Elevation Difference." Furthermore, the longest span always requires the largest offset to correct for a given difference in horizontal tension ($H_o - H$, Fig. 14).

Use in the Field

The form used to submit insulator offsets and sag corrections to the field is shown in Fig. 16.

In any sagging operation, with or without offsets, it is important to use ball or roller bearing sheaves properly maintained and lubricated to hold friction to a minimum. The quality of sheaves and the experience of the crew in working the conductor to balance out sheave friction have much to do with the length of conductor that can be properly sagged. Furthermore, all conductors in a sag-section should be treated uniformly with respect to tensions applied and duration of these tensions during stringing, prestressing, and final sagging. The average and maximum lengths of conductor sagged by BPA for NESC heavy loading are as follows:

Construction	Kv	Average Miles	Average Spans	Maximum Miles	Maximum Spans
Wood pole	115	1.5	13	2	17
Steel	230–345	2.5	13	4	20

The conductor is sagged in control spans at or near each end of the sag-section but not less frequently than at 1-mile intervals. Where possible, the longer, more level spans are selected for control spans. Sagging and plumb-marking of the conductor should be done the same day as overnight temperature changes and winds may move the conductor over the sheaves. After plumb-marking, the suspension strings may be attached to the conductor at any time, measuring the offsets from the plumb-mark as required.

Insulator Sags in Short DE Spans

In short DE spans, especially for 230 kv and higher construction where DE insulator string assemblies are quite heavy, allowance should be made for the sag of the DE assembly. BPA includes this increase in sag caused by the DE assemblies (rise of DE assemblies minus rise of conductor in the same length, $d_s - d_c$ shown in Fig. 17) whenever its value is 5% or more of the conductor sag. The method of computation is shown in Fig. 17. This same method may be used to figure sags and tensions for a conductor with concentrated loads such as wave traps, catenary lights or markers, taps, etc.

The conductor in a DE span is usually brought to sag with a DE assembly attached on one end only. To allow for the reduction in sag that will occur when a DE assembly is attached to the other end of the conductor, the conductor sag is actually increased by approximately twice the value of sag increase ($d_s - d_c$). For example, when the conductor sag should be increased by 1 foot with both DE assemblies in place, the increase during sagging operations with the DE assembly on one end only should be approximately 2 feet. This extra foot of sag during sagging operations (or 2 feet total increase) may be considered as providing the additional length ($L_s - L_c$) shown in Fig. 17.

Slack in Short Dead End Spans

Experience has shown that it is desirable to provide at least 3 inches of slack in a DE span for attaching the conductor and DE assemblies to the supporting towers. Normally this amount of slack can be provided and adequate clearance maintained in short DE spans, such as approaches to substations, by increasing the sag. Furthermore, short spans should usually be slacked to prevent excessive cold weather tensions. Turnbuckles or similar arrangement should be included in the DE hardware when

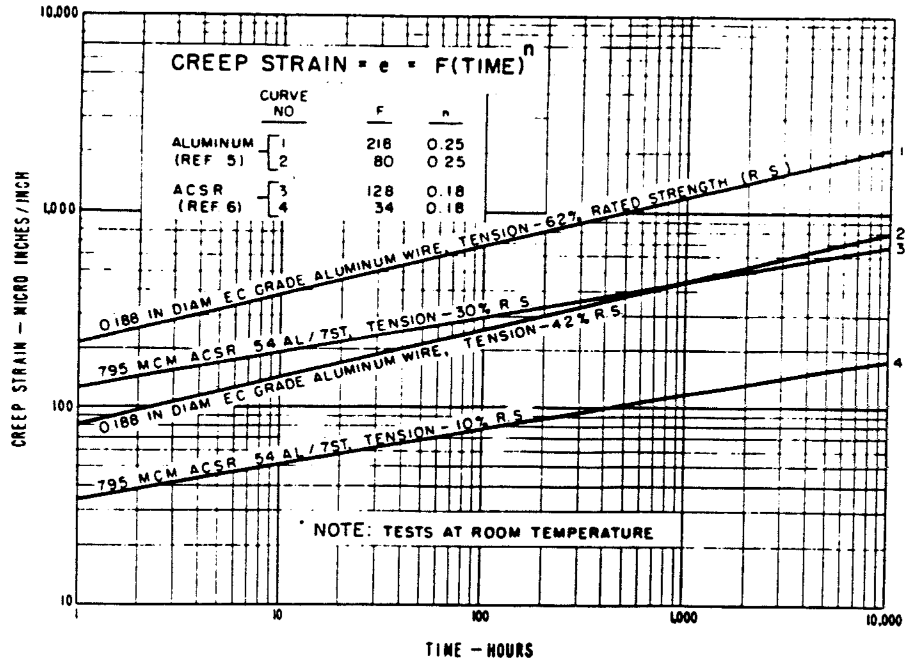


Fig. 18. Typical creep strain versus time curves

design clearance will not permit adequate slack for final hookup.

Conductor Creep

Permanent sag increase of a bare conductor from nonelastic stretch is caused by:

1. Short time loading such as from ice and wind loads in which the difference of initial and final modulus of elasticity is involved.
2. Long time loading—commonly called creep.

For a given time, conductor creep increases with increase in tension and temperature. It will also vary with type of material and stranding. For the same conditions, creep will be greatest in aluminum conductors, followed by copper, and then by steel conductors. The creep for ACSR normally falls between that for aluminum and copper conductors.

Initially creep is quite rapid with a rapid decrease in creep rate. Strand setting contributes a considerable part of the total conductor elongation during this period. This primary creep is followed by a second stage in which the creep and decrease in creep rate are both very slow. With sufficiently high tension and temperature the second stage can be followed by a rapid increase in creep rate to final conductor failure.

Experimental creep strain versus time data at normal tensions and temperatures closely follow a straight line on a log-log scale as shown in Fig. 18.[4-6] Accordingly, creep strain (e) can be represented by equation:

$$e = F(\text{time})^n \qquad (16)$$

F = constant. It is the value of creep strain at unit time (1 hour for time scale in hours, etc.) for a given slope curve.

n = constant that controls the slope of the creep curve.

This straight line plot on log-log scale permits accurate extrapolation of creep from relatively short time tests. However, ASTM (American Society for Testing Materials) designation *E 139-58T* recommends that test data cover approximately 10% of the period for which creep is to be determined.

Tests on various metals at a given temperature[5] showed that increasing creep tensions shifted the creep strain versus time curves vertically upward with no appreciable change in slope. In other words, F in equation 16 increased with tension and n remained unchanged as shown for the curves in Fig 18. More recent tests[6] on ACSR and stranded all-aluminum conductors showed that this substantially constant slope condition held for tensions varying from 10% to 60% RS (rated strength) of the conductors. Furthermore, the creep strain versus time curve for tension at 20% RS had substantially the same slope for room temperature (67 F to 75 F) as for 200 F. Thus a convenient creep table, independent of conductor tensions and normal operating temperatures, can be prepared for extrapolating creep measured in the field to any time period desired such as 10, 20, 30 years, etc. (see Table III). This assumes that the variations in conductor tension and temperature during period of creep measurements will be typical of the variations during the extrapolated period. However, creep will cause some reduction in the conductor tension depending on DE or ruling span length and H/w. The effects of this reduced tension on the extrapolated value of creep can be evaluated.

Conductor creep in a given transmission line will be dependent on the everyday stresses and operating temperatures occurring. These will vary with time and location. Creep studies for different conductor types are being made in selected spans by BPA (presently on ACSR of two, three, and four layers of aluminum). Two adjacent suspension spans close to ruling span length are excellent for such creep studies. Spans are selected in areas subject to infrequent icing to simplify the problems of evaluating that part of conductor elongation caused by icing loads. Readings should be taken after any severe icing to isolate these readings from subsequent creep readings. Sag, conductor temperature, movement of supports, and wind velocity and direction are measured.

It is most convenient to express the creep or permanent sag increase in terms of a temperature increase above ambient. Such creep measurements for ACSR Pheasant with everyday stresses averaging 20% RS, showed creep in the order 15 F temperature increase for 1,000 hours. On the basis of Table III for 795-MCM (thousand circular mils) ACSR conductor of similar stranding and per

Table III. Creep Ratio

Time for Creep Measurement	795-MCM ACSR, 54 AL/7 ST			397.5-MCM All-Aluminum, 19 ST		
	10 Years	30 Years	50 Years	10 Years	30 Years	50 Years
Hours, 1,000	2.20	2.70	2.95	2.40	3.00	3.30
Years, 1	1.55	1.90	2.05	1.60	1.95	2.15
2	1.30	1.60	1.75	1.35	1.70	1.85
3	1.25	1.50	1.65	1.30	1.60	1.75
5	1.15	1.40	1.50	1.15	1.45	1.60
10	1.00	1.20	1.35	1.00	1.25	1.35
30		1.00	1.10		1.00	1.10
50			1.00			1.00

Creep for the time listed at head of Columns 2 through 7 is approximately equal to the creep measured for the time listed in Column 1 multiplied by appropriate creep ratio. Time (Column 1) versus creep ratio plots as a straight line on log-log scale.

cent steel the creep in 10 and 30 years would be (2.20) (15) or 33 F and (2.70) (15) or 41 F respectively. However, data would be desirable for a longer time period for such extrapolation.

Allowance for creep during sagging can be made either by including the creep correction in the stringing charts or by reading the sag from an uncorrected chart at ambient temperature minus the established temperature correction for creep.

Prestressing For Controlling Creep

Controlled laboratory creep tests on stranded aluminum alloy conductor raise some question on the long time benefits of prestressing. Prestressing reduced subsequent creep during these tests as would be expected; however, the slope of the creep curve was increased. This would indicate that the advantage of prestressing would gradually become lost. It is interesting to note from reference 5 that cold working also increases the slope of the creep curves for aluminum and aluminum alloys. As mentioned previously BPA prestresses primarily to stabilize the conductor to the virtual initial modulus of elasticity curve.

Conclusions

Refinements of sag-tension computations in the office can produce no better end results than permitted by the approximations and tolerances in field measurements. Therefore, an important part of the design process is to provide simple, practical instructions and adequate controls for these measurements in the field.

Summary of Nomenclature

A,B,C = horizontal, vertical, and slope distance between supports, Fig. 1
A_r = ruling span
E,E_s = modulus of elasticity, conductor and steel core
H = horizontal component of conductor tension, Fig. 1
H_o = horizontal component of conductor tension in suspension clamp, Fig. 13
H_e = equivalent horizontal component of conductor tension in sheaves, Fig. 13
H' = corrected horizontal component of conductor tension in sheaves, Fig. 13
$K = w^2A^2/H_o^3$ = change in slack or offset in inches for a span per pound change in horizontal tension, Fig. 14
L = conductor length between supports, Fig. 1
L_c = conductor length replacing L_s in catenary, Fig. 17
L_s = dead end assembly length, Fig. 17
MWT = maximum working tension
RS = rated strength
S = sag = distance in vertical plane measured from line between supports to the point on the conductor with B/A slope, Fig. 1
S_1 = sag distance at point of tangency of conductor and line of sight, Fig. 10
T = conductor tension at any point on catenary, Fig. 1
T_e = value of T for the catenary that satisfies Hooke's law
T_m = value of T for the catenary at the support
$Z = wx/H$ = parameter
a,a_s = cross section area, conductor and steel core
d_c = vertical rise of L_c, Fig. 17
d_s = vertical rise of L_s, Fig. 17
e = creep strain, Fig. 18
h = vertical distance from conductor support to transit for measuring sag, Fig. 9
ℓ = conductor length measured from low point of catenary to any point x,y; Fig. 1
ℓ_u = unstressed length of ℓ
t = vertical distance from conductor support to target for measuring sag, Fig. 9
w = weight of conductor per unit length, general; Fig. 1
$w_1 = w$ for bare conductor, no wind, in pounds per foot
$w_2 = w$ for conductor and ice, no wind, in pounds per foot
$w_3 = w$ in plane of resultant for conductor, ice, and wind in pounds per foot
x,x_1,x_2,y,y_1,y_2 = catenary co-ordinates, Fig. 1

References

1. ACSR Graphic Method for Sag-Tension Computations (book), Theodore Varney. Aluminum Company of America, Pittsburgh, Pa., 1927.

2. Transmission Line Catenary Calculations, D. O. Ehrenburg. *AIEE Transactions*, vol. 54, 1935, pp. 719–27.

3. Sag Calculations by the Use of Martin's Tables (book), James S. Martin. Copperweld Steel Company, Glassport, Pa., 1931.

4. Mechanics of Creep for Structural Analysis, Joseph Marin. *Proceedings*, American Society of Civil Engineers, New York, N. Y., vol. 68, 1942, pp. 719–35.

5. A Method of Analyzing Creep Data, R. G. Sturm, C. Dumont, F. M. Howell. *Transactions*, American Society of Mechanical Engineers, New York, N. Y., vol. 58, 1936, pp. A62–A66.

6. Effects of Creep on Sags and Tensions of Overhead Conductors, George H. Phillips. Aluminum Company of America, Pittsburgh, Pa., 1958.

Discussion

H. H. Rodee (Aluminum Company of America, Pittsburgh, Pa.): The author has made an excellent contribution to an understanding of the problems involved in the calculation of sags and tensions of ACSR and application of the results to field use in the installation of transmission line conductors.

The method described is essentially the same as used by Alcoa and utilizes the stress-strain curve for the conductor in lieu of numerical values for modulus of elasticity. The tension scale of the tension-strain curves and catenary function curves is in pounds, whereas our usual practice is to use pounds per square inch. The advantage of using tension in pounds is that the values are obtained directly, but the disadvantage is that tension-strain diagrams are required for each individual conductor size. When the scale is in pounds per square inch, the same diagrams and also the same catenary function curves for bare conductor may be used for all conductor sizes of the same type of stranding; i.e., same proportions of steel and aluminum. This greatly reduces the number of curves required for making calculations for a variety of conductor sizes. The method described by the author is convenient for those dealing with a small number of conductors, but pounds per square inch seems more practical for those working with a large number of conductors.

The author calls attention to two factors that will cause permanent increase in sag from nonelastic stretch. One is increase in load as from ice and wind, and the other is a gradual nonelastic stretch over a long period of time with no significant increase in load. This is commonly referred to as creep. The permanent increase in sag resulting from these causes is not cumulative, and whichever produces the greater sag will control the design.

In the case of a transmission line conductor, certain factors that influence creep are to some extent self-corrective. The rate of creep increases as the tension increases and conversely decreases as the tension is reduced. Since the tension decreases as sag increases as a result of creep, this condition tends to correct itself. The rate of creep also increases as the temperature increases, but in the case of a transmission line conductor the tension decreases as temperature increases, and again the increased rate because of higher temperature is to some extent counteracted by the reduced tension.

In the case of ACSR, the creep results primarily from creep of the aluminum stands; and, since the proportion of the load carried by the aluminum decreases as temperature increases because of difference in coefficient of expansion, the rate of creep at higher temperature will be further reduced.

Span length also has an influence on creep. As the length of conductor increases because of creep, the tension will decrease. The rate at which the tension decreases will be greater in short spans than in long spans and the rate of creep will, therefore, be reduced more rapidly in short spans. Consideration of creep in design is therefore more important on long spans than on short spans.

B. M. Pickens (Anaconda Wire & Cable Company, Hastings on Hudson, N. Y.): In connection with creep we would like to know if BPA has checked the sags in any of their lines that have been installed for several years to see if the sag has increased more than expected?

We note that the prestressing practiced is to stabilize the conductor to the virtual initial modulus of elasticity. Approximately what tension is used, in per cent of rated strength, and for what length of time is the conductor maintained at this tension?

The graphical method for calculating sag-tension values is very complete and shows the difference between the tension causing elongation of the conductor which is an aver-

Table IV

Loading Conditions						
Ice, inches	1/2	0	0	0	0	0
Temp., F	32	0	30	60	90	120
Final Sags						
Fig. 5	80.80					83.90
Computer	80.63					83.60
Initial Sags						
Fig. 5		72.10	74.41	76.68	79.96	81.12
Computer		72.02	74.33	76.58	78.80	80.96

age tension, and the tension at the support which is a maximum tension. They are not the same. This is sometimes overlooked in calculating long spans. As can be seen from the several curves, Figs. 3, 4, 5, and 6 the graphical solution requires a lot of work. The use of a digital computer can reduce the time required for some of the computations. Using the program described in an AIEE paper,[1] the results given in Table IV were obtained for an 1,800-foot span which can be compared with results shown in Fig. 5 of this paper.

We believe this is a very close check between a graphical and a computer method, particularly since the stress-strain curves used were from different tests. The computer solution can be made in a few minutes while the graphical solution probably takes hours.

REFERENCE

1. SAG-TENSION CALCULATION PROGRAM FOR DIGITAL COMPUTER, B. M. Pickins. *AIEE Transactions*, pt. III (*Power and Apparatus and Systems*), vol. 77, 1958 (Feb. 1959 section), pp. 1308–15.

L. H. J. Cook (B. C. Engineering Company, Ltd., Vancouver, B. C., Canada): The author is to be congratulated for a very comprehensive résumé of the problems of conductor stringing and sagging, and the solutions to these problems. Included in his paper are a family of curves showing the relationship between time and conductor creep for various designs. I would be most interested to learn whether he has any similar data on the relationship between time and conductor creep for varying conductor temperatures.

The author's comments regarding the correct relationship between design procedures and accuracies and the approximations and tolerances in field measurements emphasizes one of the most critical problems of transmission design. My company's procedure has been to produce complete sagging data well ahead of stringing. The data specify the spans in which the sags will be measured and also defines the temporary snubbing positions. This method requires a considerable amount of staff work and planning but has been found to produce good results. Computer programs reduce the engineering time to a minimum.

The problems outlined by the author become compounded when applied to the bundle conductors. Small variations of a few inches between the sags of the conductors of one bundle could lead to undesirable results.

During the early stages of stringing the Rosedale-Creekside 360-kv transmission line problems were encountered in maintaining equal sags between the two conductors of the bundle. This was principally due to unequal creep in the two conductors immediately after stringing. This problem was solved by prestressing the conductors by one of the following methods.

METHOD 'A'

The conductors were pulled up to 20% above the required stringing tension and left at this tension for a minimum of 15 minutes. The conductors were then slacked off, sagged, and clipped in.

METHOD 'B'

The conductor was pulled up to stringing tension and was left in the sheaves at this tension overnight. The following morning the conductor was checked, adusted if necessary, and then clipped in.

Method 'B' was offered as a convenience to the stringing contractor in the event that he completed his pull towards the end of the working day.

F. W. DeMoney (Kaiser Aluminum & Chemical Corporation, Spokane, Wash.): The author has made a significant contribution to the general field of developing proper sag-tension computations. The techniques employed in field measurements are developments necessary to verify the theoretical calculations.

The author is to be congratulated for recognizing the creep of conductors as a significant factor contributing to sag. This recognition of creep is reassuring to those working to develop reliable creep information on conductors. Because we are interested in this field of conductor creep, we have the following questions:

1. How will laboratory-developed creep data obtained under constant tension and constant temperature conditions be utilized in sag-tension calculations?
2. Can extrapolated values of creep strain for 10, 30, and 50 years, which have been obtained by extrapolating the 1,000-to-2,000-hour laboratory creep data, be used with confidence in this application?

Answers to these questions could aid materially in the design of creep experiments on electrical conductors, and in the analysis and interpretation of creep results.

P. F. Winkelman: I appreciate the interest shown and valuable comments made by the discussers.

Mr. Rodee presents an excellent analysis of the factors influencing creep in a transmission line conductor. They are basic and help to rationalize the problems involved. The permanent increase in sag of the bare conductor and corresponding reduction in tension from ice and wind loads involving a change from initial to final modulus of elasticity will cause some decrease in creep rate. However, subsequent creep will cause additional permanent increase in sag. The cumulative effects of both types of nonelastic stretch must be considered in maintaining minimum code clearances.

With reference to Dr. DeMoney's questions, the paper describes one means of utilizing controlled laboratory creep data to estimate the actual creep to be expected in a transmission line in which there is no control of the variables concerned. ASTM designation *E139-58T* may be used as a guide for the duration of creep tests. We believe that field measurements in selected spans with their variations in tension and temperature should be taken for a longer period of time than laboratory measurements on creep.

For long-time problems such as creep, more data are needed on the different types and combinations of conducting materials including the aluminum alloys. Also of considerable value would be more laboratory data on: 1. the amount and duration of strand setting at various tensions and for different types of stranding, 2. the amount of strand setting reoccurring when using old conductor that is rereeled and then used in a new location, and 3. the effects of prestressing at various tensions and hold periods on subsequent creep of single and multiple strand conductors. We value the assistance and information received from conductor manufacturers in the past on problems such as these.

The relationship between time and conductor creep for 20, 40, and 60 per cent of rated strength at both room temperature and 200 F is given for an all-aluminum stranded conductor in reference 6 of the paper and may provide an answer to the inquiry in Mr. Cook's discussion. However, it seems desirable to have such data for a period longer than 42 days.

In answer to Mr. Pickens' question, we are now correcting for creep in all our stringing for both medium and heavy loadings on basis of sag checks made in the past several years.

The International Business Machines Corporation *650* is now being used for computing insulator offsets and sag corrections by the long form method. We are also working on programs in the "Fortran" language for solving other conductor problems including one for computing sags and tensions for use in economic studies and preparation of clearance templates. However, the graphical method will still be used in preparing sag charts for field sagging. The complete files of catenary function curves, stress-strain curves, and sag charts, once they have been prepared, are repeatedly utilized as the system is developed.